State-of-Charge Estimation

TSFS19 Battery Systems - Lecture 5

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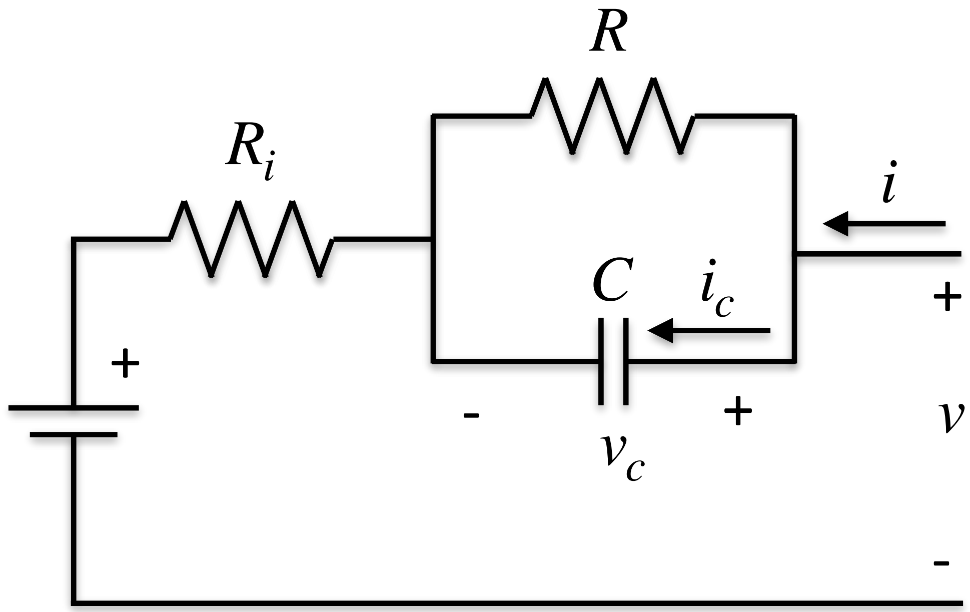
Where we are

- Previous lectures:
  - ECM
    - Identify battery capacity from measurements.
    - Identify the open circuit voltage as a function of SOC from measurement data.
    - Identify impedance parameters from dynamic operation.
- Today's lecture:
  - Utilize ECMs for state-of-charge estimation. (Lab 2)



SOC-Estimation

SOC-Estimation

- An estimate of all battery-pack cells' SOC is an important input to balancing, energy, and power calculations.
- While we might be interested in estimating the entire battery-model state, we first focus on estimating state-of-charge only.
  - We'll see some simple methods that lack robustness.
  - Then, we examine methods that estimate the entire battery-model state, enabling some more advanced applications.
- The model estimated in the previous lecture with on RC-link will be used.
  - The OCV-curve and the parameters are identified on other data than used for SOC estimation.

Benefits of Accurate SOC Estimates

- **Life:** Prevents overcharging or over-discharging, protecting battery cells from damage and extending their lifespan.
- **Performance**: Enables full utilization of battery capacity, reducing the need for overly conservative usage.
- **Density**: Allows smaller, lighter battery packs by using the battery efficiently within design limits.
- **Economy**: Reduces costs through smaller systems and fewer warranty claims due to increased reliability.

Recall Definition of State-of-Charge

- State of charge (SOC):

$$\mathrm{SOC} = \frac{\text{available charge}}{\text{total charge capacity}} \cdot 100\,\%$$

- Percentage of charge in the battery/cell
  - 100% full
  - 0% empty

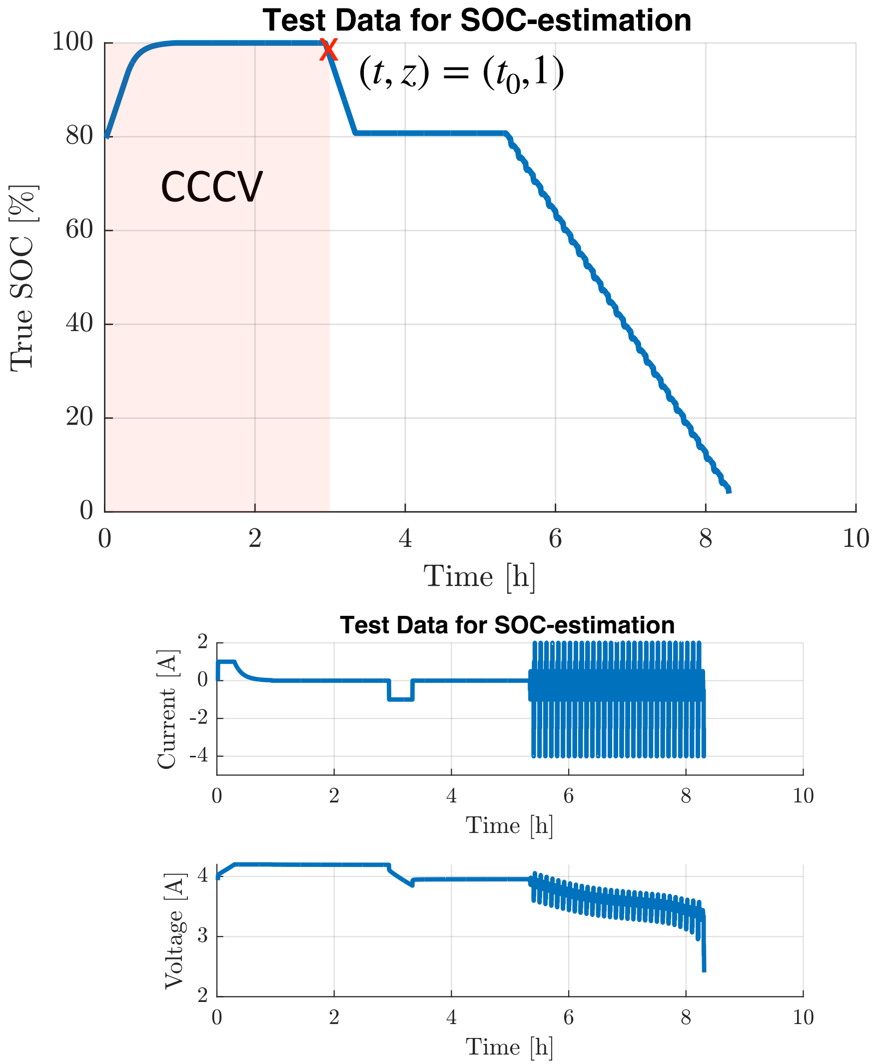
Some Approaches to Estimate State of Charge

Approximation of the True SOC

- **Assumption**: Actual SOC is 100% at the end of a CCCV (Constant Current Constant Voltage) charging cycle.
- **Total Capacity (Q)**: Identified from full charge/discharge cycle data.
- **"True" SOC Computation:**
  - Method: Coulomb counting.
  - Equation:

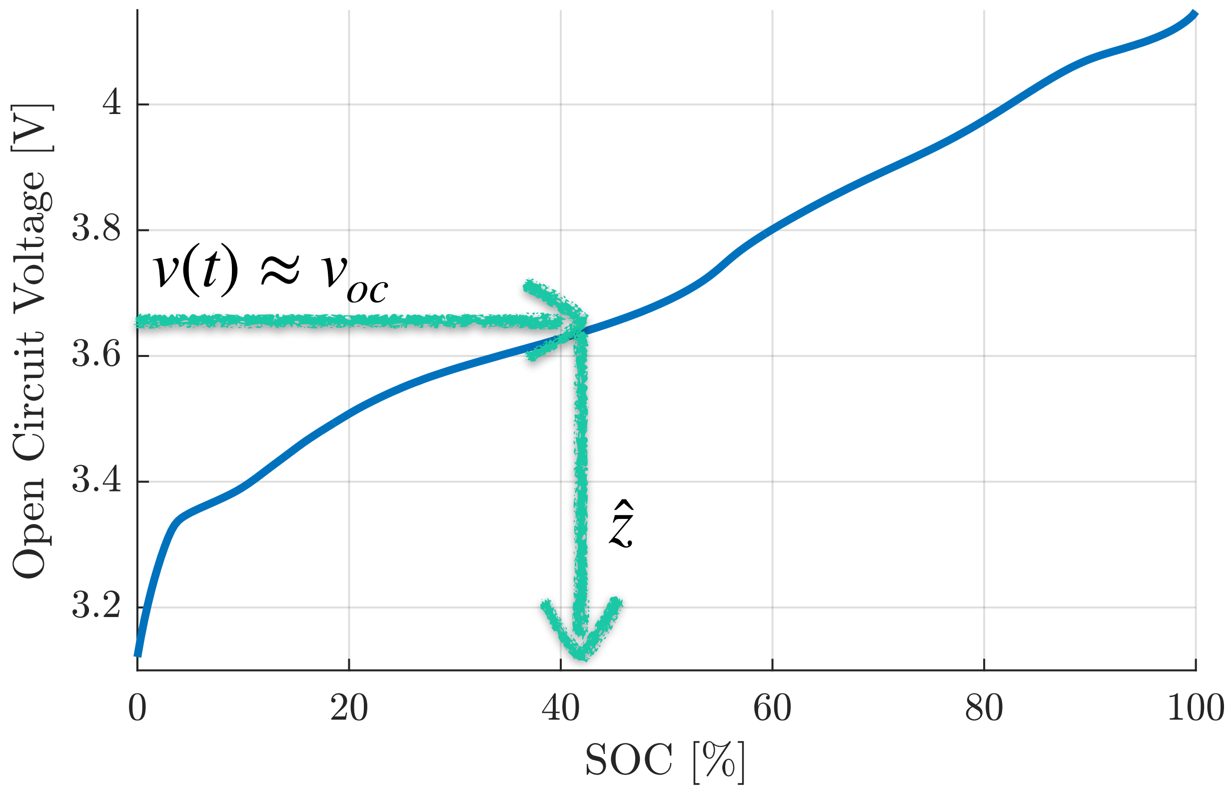
$$z(t) = 1 + \frac{1}{Q}\int_{t_0}^{t} i(t)d\tau$$

  where t_0 is the time just after CCCV ends, and $i(\tau)$ is the current.
- This approximation of SOC is only valid for **short-time series**.



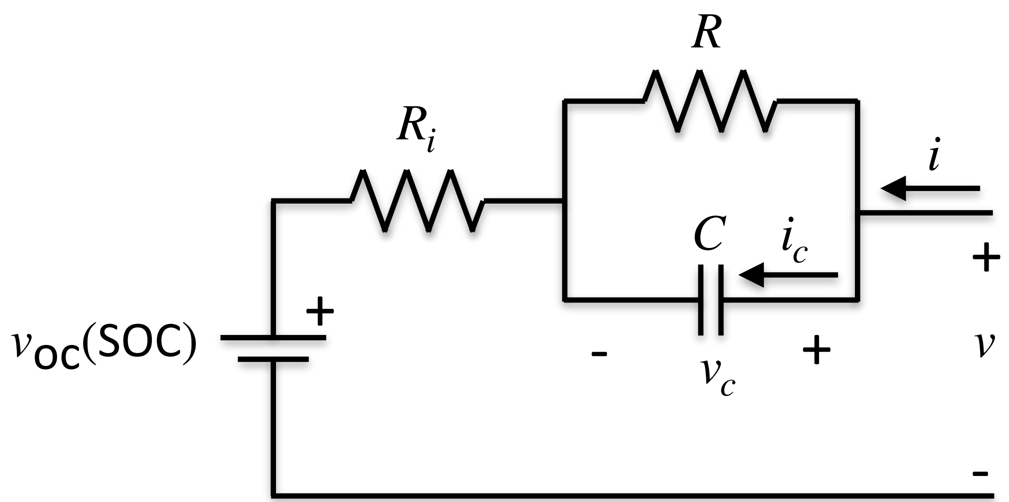
SOC Estimation via Voltage-Based Method

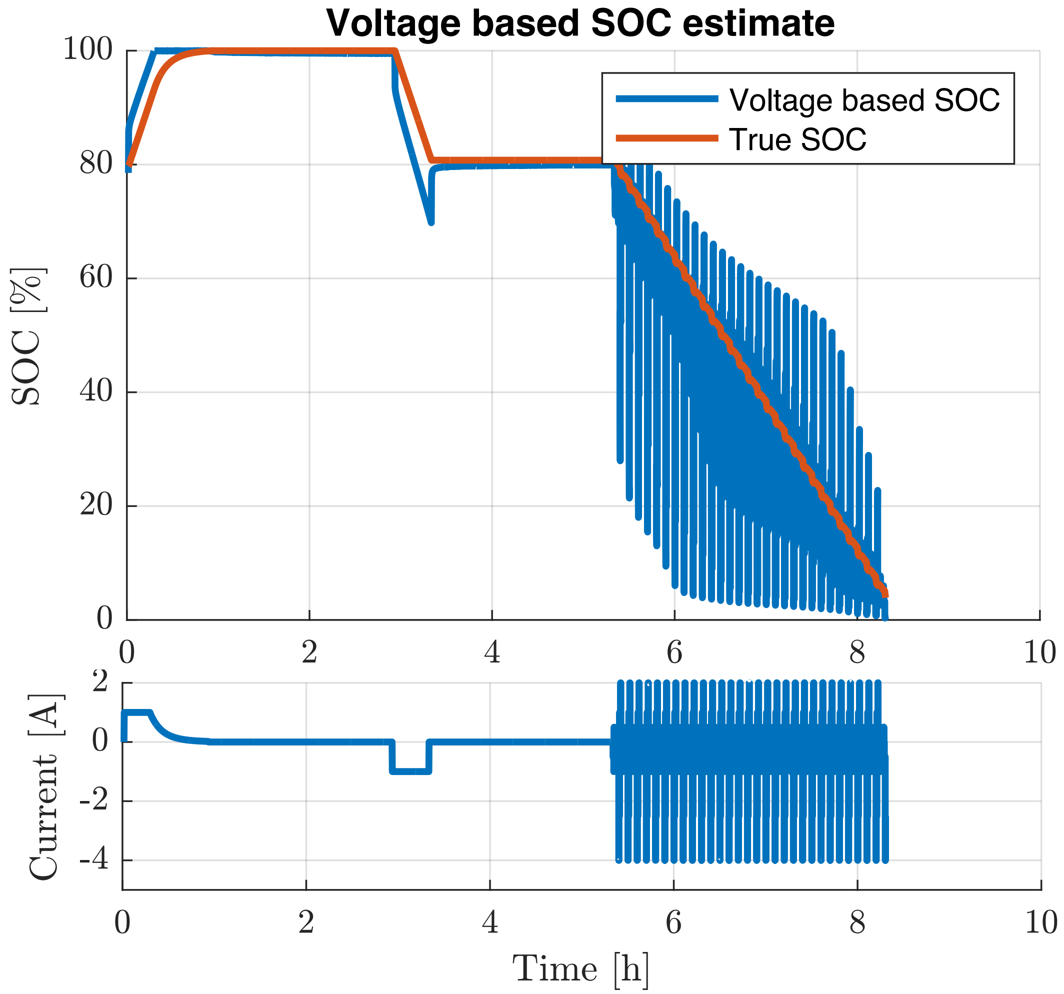
- Measure cell terminal voltage under load $v(t)$
- Assume $v(t) \approx v_{oc}$
- Use the OCV-function
$$v_{oc} = \mathrm{OCV}(z)$$
to infer the SOC:
$$\hat{z} = \mathrm{OCV}^{-1}(v_{oc}) \approx \mathrm{OCV}^{-1}(v)$$
- Misses effects of resistive losses, diffusion voltages
- Wide flat areas of OCV-curve decrease the accuracy of the estimate



Poor, Voltage-based Method for SOC Estimation

- Voltage-based SOC estimate works for the no-current case with a relaxed battery.
- RMSE SOC: 9.18 %



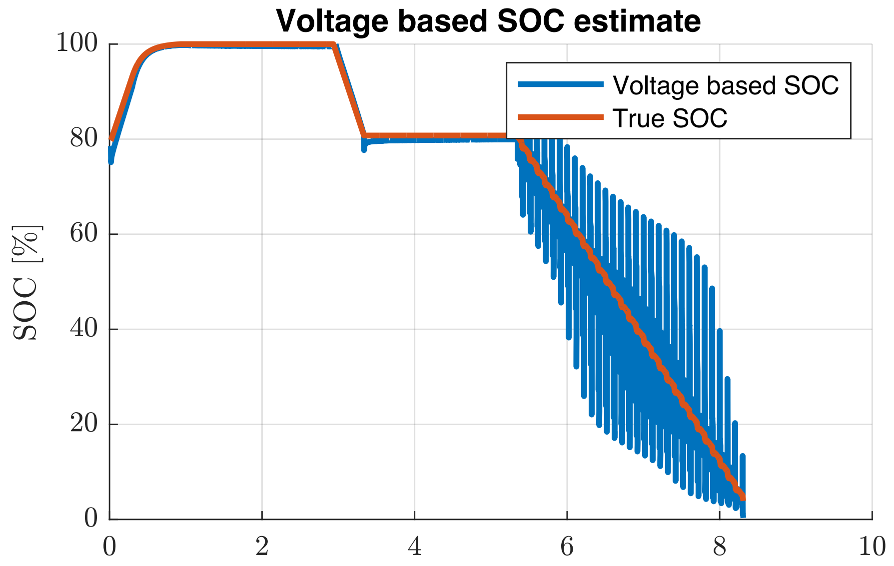


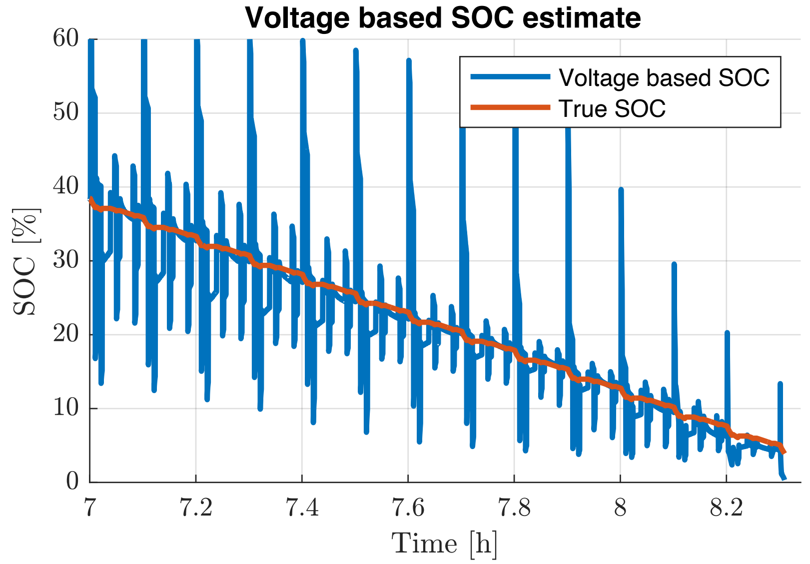
SOC Estimation based on Thevenin Equivalent Circuit

- Assume the model $v(t) = v_{oc}(t) + i(t)R$ where R is total cell resistance.
- SOC-estimation based on Thevenin equivalent model:

$$z(t) = \mathrm{OCV}^{-1}(\underbrace{v(t) - i(t)R_0}_{=\hat{v}_{ocv}})$$

  - Thevenin equivalent circuit model works for constant currents.
  - RMSE SOC: 3.30 %





Poor, Current-based Method to Estimate SOC

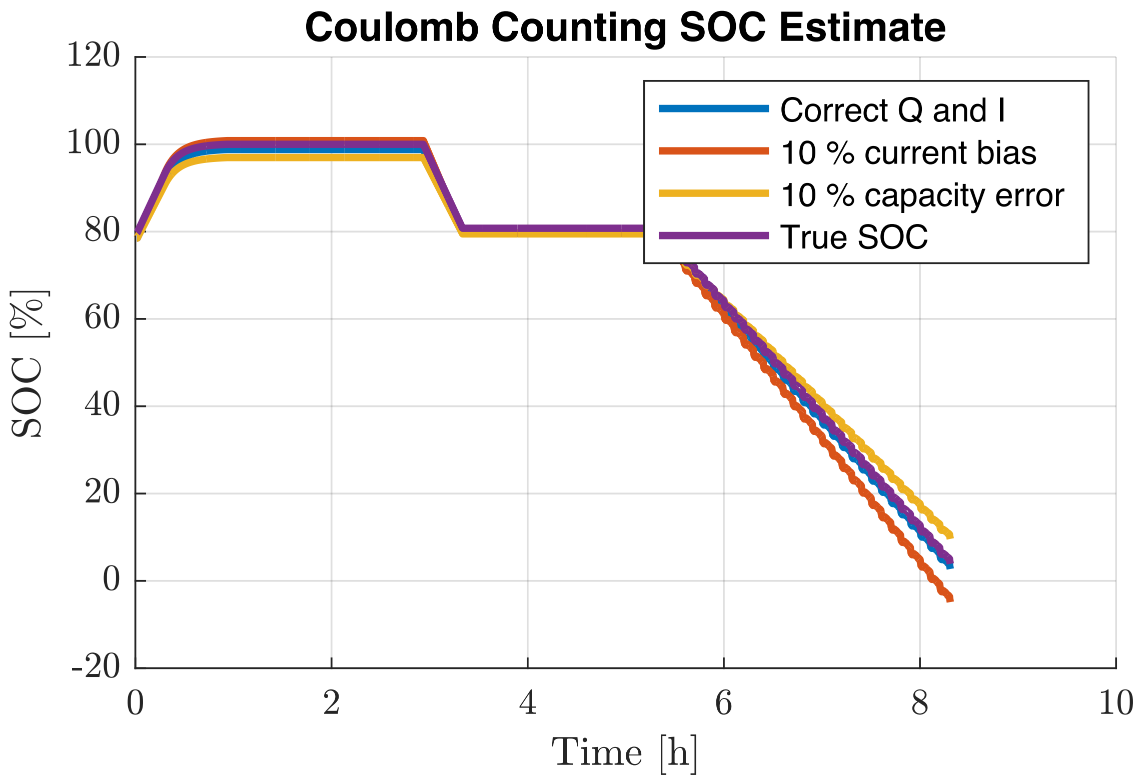
- Coulomb counting keeps track of the charge in and out of the cell

$$z(t) = z(0) - \frac{1}{Q}\int_0^t i(t)dt$$

- A time-discretized form

$$z_{k+1} = z_k - \frac{\Delta t_k}{Q} i_k$$

- Okay for short periods of operation when initial conditions are known or can frequently be "reset".
- Subject to drift due to current sensor's fluctuations, current-sensor bias, incorrect capacity estimate, and other losses
- Uncertainty/error bounds grow over time, increasing without bound until the estimate is "reset".
- RMSE SOC: 1.19 % 3.30 % 2.60 %



Filter-based approaches to combine the voltage and current-based information!

Model-based State-Estimation

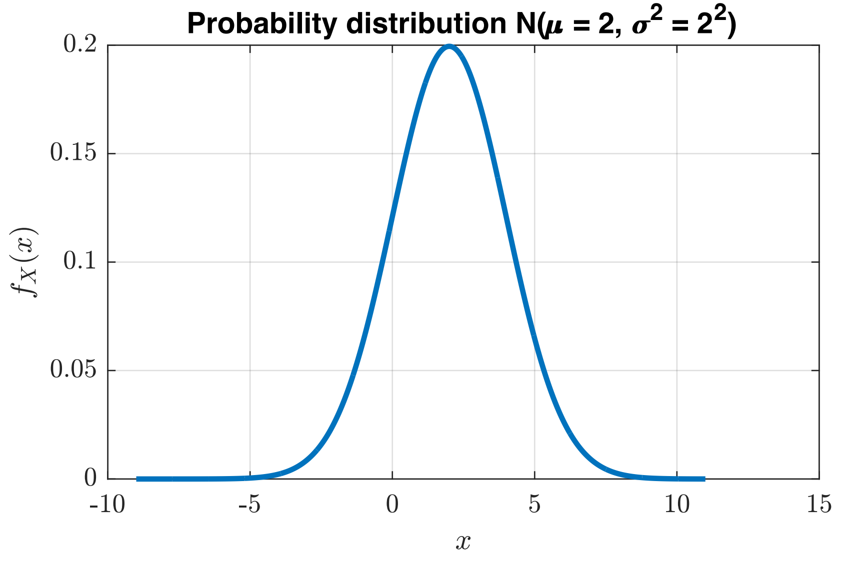
Extended Kalman Filter

Using Extended Kalman Filter (EKF) for SOC Estimation

- The EKF is an advanced technique used to estimate the State of Charge (SOC) of batteries, especially under dynamic conditions.
- It combines the information of current and voltage measurements to estimate the SOC.
- The EKF is a **probabilistic framework** for estimating the SOC of batteries, allowing for **control of the trade-off** between voltage and current measurements by assigning uncertainties.
- Batteries exhibit nonlinear characteristics, especially in the OCV-function.
- EKF is an extension of the Kalman Filter, designed to handle nonlinear systems.

Some Probability Theory

- $X \sim N(\mu, \sigma^2)$: A random variable X is normal distribution with mean μ and variance σ^2.
- Expected value: $\mathrm{E}(X) = \mu$
- $\mathrm{Var}(X) = \mathrm{E}[(X-\mu)^2] = \sigma^2$



- $X \sim N(\bar{x}, \Sigma_{\tilde{X}})$: A random **vector** X.
- Expectance value: $\mathrm{E}(X) = \bar{x}$
- Correlation matrix $\Sigma_X = \mathrm{E}(XX^T)$
- Covariance matrix $\Sigma_{\tilde{X}} = \mathrm{E}((X-\bar{x})(X-\bar{x})^T)$

- Covariance vs variance:

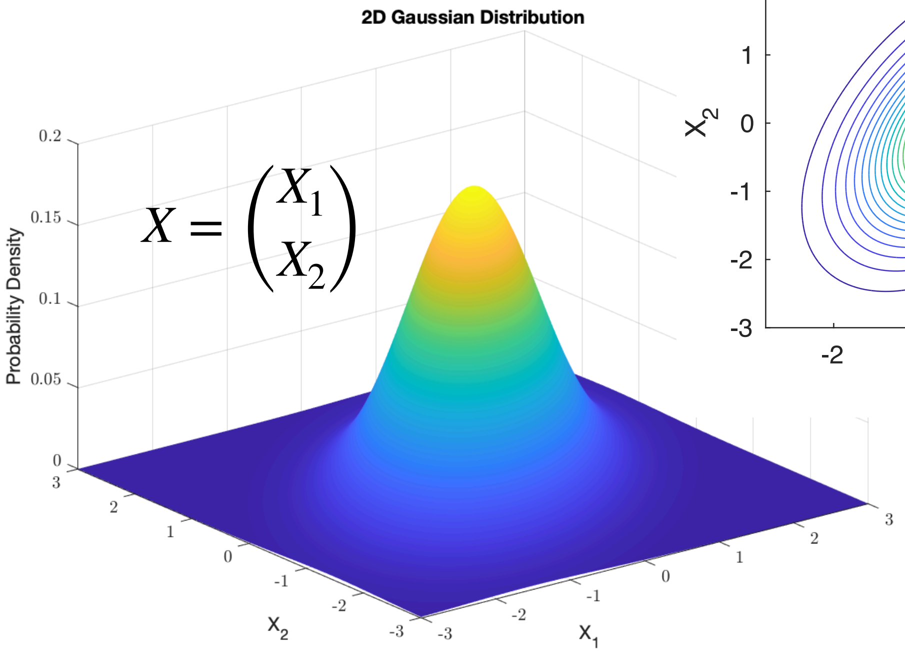
$$\Sigma_{\tilde{X}} = \begin{bmatrix} \sigma_{X_1}^2 & \rho_{12}\sigma_{X_1}\sigma_{X_2} \\ \rho_{12}\sigma_{X_1}\sigma_{X_2} & \sigma_{X_2}^2 \end{bmatrix}$$

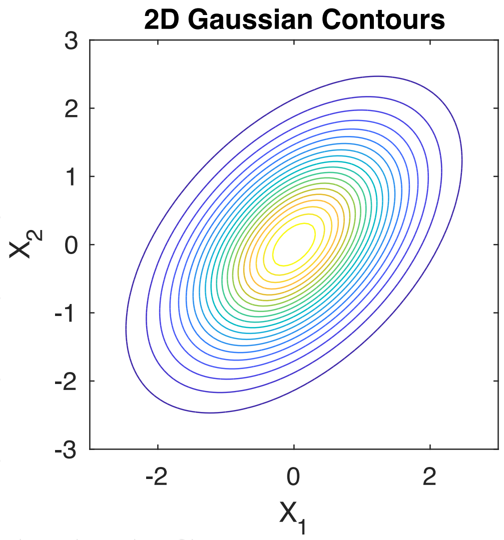
Correlation. Uncorrelated if 0

Variance

where ρ_{12} is the correlation coefficient measuring the linear dependence between X_1 and X_2. $|\rho_{12}| \leq 1$

Some Probability Theory





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Variance

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Kalman Filter

Key Concepts of the Kalman Filter

- The Kalman filter is designed to estimate the state of a system, which could include variables like SOC in the battery case.
- The system is typically described by a state vector x_k that evolves.

System Model

The Kalman filter assumes the system follows a linear dynamic model, described by two main equations.

The input u_k has been omitted for simplicity, but it is straightforward to include.

State Equation

$$x_{k+1} = F_k x_k + G_k w_k$$

- F_k: State transition matrix (describes how the system evolves).
- Control matrix (describes the effect of control input
- w_k: White process noise with 0 mean $E(w_k) = 0$ and covariance matrix $E(w_k w_k^T) = Q_k$ (uncertainty in the model).

Measurement Equation

$$y_k = H_k x_k + v_k$$

- H_k: Measurement matrix (relates the state to the observed measurement).
- v_k: White measurement noise with 0 mean $E(v_k) = 0$ and covariance matrix $E(v_k v_k^T) = R_k$ (uncertainty in the measurements).

Kalman Filter

The goal is given

- an initial state x_0 with covariance matrix $P_0 = E[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T]$ describing the uncertainty of the initial state,
- inputs $u_1, \ldots, u_k$, and outputs $y_1, \ldots, y_k$

to estimate the state x_k of the system as accurately as possible, i.e,

- the most likely state estimate $\hat{x}_k = \mathrm{E}(x_k)$
- with the smallest possible covariance matrix $P_k = E[(x_k - \hat{x}_k)(x_k - \hat{x}_k)^T]$ describing the uncertainty of the state estimate.

Battery context

Given an initial SOC

+

Current och voltage measurements

Optimal SOC estimate with a description of the uncertainty.

Kalman Filter Procedure

The filter is often written as:

1. Initialize the filter with apriori informatio

$$\hat{x}_{0|-1} = x_0, \quad P_{0|-1} = P_0 \text{ where}$$

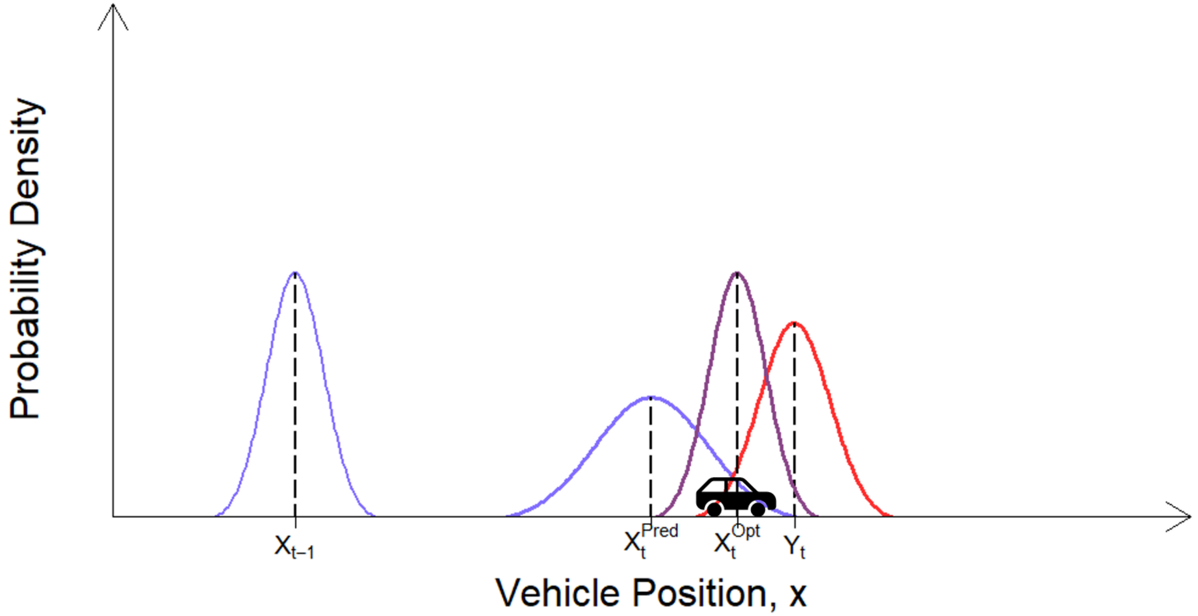
2. Measurement update; take a new measurement y_t and

$$(\hat{x}_{t|t-1}, P_{t|t-1}) \quad \rightarrow \quad (\hat{x}_{t|t}, P_{t|t})$$

3. Time update; proceed to next time-step

$$(\hat{x}_{t|t}, P_{t|t}) \quad \rightarrow \quad (\hat{x}_{t+1|t}, P_{t+1|t})$$

Iterate from step 2.



Kalman Filter, the equations

- If you look up the equations, they might look like
- Measurement update

$$K_k = P_{k|k-1}H_k^T(H_kP_{k|k-1}H_k^T + R_k)^{-1}$$

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k(y_k - H_k\hat{x}_{k|k-1})$$

$$P_{k|k} = P_{k|k-1} - K_kH_kP_{k|k-1}$$

- Time update:

$$\hat{x}_{k+1|k} = F_k\hat{x}_{k|k} + B_ku_k$$

$$P_{k+1|k} = F_kP_{k|k}F_k^T + G_kQ_kG_k^T$$

- The measurement equation is non-linear in the battery case and a non-linear extension is needed.

System model

$$x_{k+1} = F_kx_k + B_ku_k + G_kw_k \qquad Ew_k = 0, \quad Q_k$$
$$y_k = H_kx_k + v_k \qquad Ev_k = 0, \quad R_k$$

Determine the Kalman gain (determines how much to trust the prediction vs. the measurement)

Correct predicted state using measurement

Adjust the uncertainty based on how much the measurement corrects the prediction

Predict the next state based on the current state and system model.

Predict the next covariance (uncertainty) based on the model and process noise.

Extended Kalman Filter (EKF)

Non-linear extension to KF needed for the battery model

Extended Kalman Filter (EKF)

- Extended Kalman Filter is a method where the same methodology is used for non-linear systems.

$$x_{k+1} = f(x_k, u_k, w_k) \qquad Ew_k = 0, \quad Q_k$$
$$y_k = h(x_k, u_k) + v_k \qquad Ev_k = 0, \quad R_k$$

Linear system model

$$x_{k+1} = F_k x_k + B_k u_k + G_k w_k \qquad Ew_k = 0, \quad Q_k$$
$$y_k = H_k x_k + v_k \qquad Ev_k = 0, \quad R_k$$

Difficulty:

- Expected value and variance in linear transformations are easy to express explicitly.
- This is not true for nonlinear transforms.

Idea:

- Compute the Kalman gain and covariance matrices by linearizing around the current state x_k.
- The problem is that we do not know x_k, instead, we linearize around our best guess $\hat{x}_k$.

Extended Kalman Filter

Non-linear system model

$$x_{k+1} = f(x_k, u_k, w_k) \qquad Ew_k = 0, \quad Q_k$$
$$y_k = h(x_k, u_k) + v_k \qquad Ev_k = 0, \quad R_k$$

Measurement Update

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k(y_k - h(\hat{x}_{k|k-1}, u_k))$$
$$K_k = P_{k|k-1} H_k^T (H_k P_{k|k-1} H_k^T + R_k)^{-1}$$
$$P_{k|k} = P_{k|k-1} - K_k H_k P_{k|k-1}$$

$$H_k = \frac{\partial}{\partial x} h(x, u)\,\Big|_{x=\hat{x}_{k|k}, u=u_k}$$

Time Update

$$\hat{x}_{k+1|k} = f(\hat{x}_{k|k}, u_k, 0)$$
$$P_{k+1|k} = F_k P_{k|k} F_k^T + G_k Q_k G_k^T$$

$$F_k = \frac{\partial}{\partial x} f(x, u, w)\,\Big|_{x=\hat{x}_{k|k}, u=u_k, w=0}$$

$$G_k = = \frac{\partial}{\partial w} f(x, u, w)\,\Big|_{x=\hat{x}_{k|k}, u=u_k, w=0}$$

To apply the filter we need to define f, $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial w}$, h, $\frac{\partial h}{\partial x}$, $\hat{x}_{0|-1}$, $P_{0|-1}$, Q_k, and R_k.

EKF-based SOC-Estimation

Preparing to Implement EKF on ESC model

State equations

- State of charge

$$z_{k+1} = z_k + \frac{\Delta t_k}{Q} i_k + w_{1,k}$$

- Capacitor voltage

$$v_{c,k+1} = (1 - \frac{\Delta t_k}{R_1 C_1}) v_{c,k} + \frac{\Delta t_k}{C_1} i_k + w_{2,k}$$

Measurement equation

$$v_k = \underbrace{\mathrm{OCV}(z_k) + v_{c,k} + R_0 i_k}_{=h(x_k, u_k)} + e_k$$

Variables

$$x_k = \begin{bmatrix} z_k \\ v_{c,k} \end{bmatrix}, \ u_k = \begin{bmatrix} i_k \\ \Delta t_k \end{bmatrix}, \ y_k = v_k, \ w_k = \begin{bmatrix} w_{1,k} \\ w_{2,k} \end{bmatrix}$$

Covariances and initial guess

$$\Sigma_{w_k} = Q = \begin{bmatrix} \sigma_z^2 & 0 \\ 0 & \sigma_{v_c}^2 \end{bmatrix}, \quad \Sigma_{e_k} = R = \sigma_v^2, \quad x_0 = \begin{bmatrix} z_0 \\ v_{c,0} \end{bmatrix}, \qquad P_0 = \Sigma_{x_0} = \begin{bmatrix} \sigma_{z_0}^2 & 0 \\ 0 & \sigma_{v_{c,0}}^2 \end{bmatrix}$$

State equation

$$\underbrace{\begin{bmatrix} z_{k+1} \\ v_{c,k+1} \end{bmatrix}}_{=x_{k+1}} = f(x_k, u_k, w_k) = \begin{bmatrix} z_k + \frac{\Delta t_k}{Q} i_k + w_{1,k} \\ (1 - \frac{\Delta t_k}{R_1 C_1}) v_{c,k} + \frac{\Delta t_k}{C_1} i_k + w_{2,k} \end{bmatrix}$$

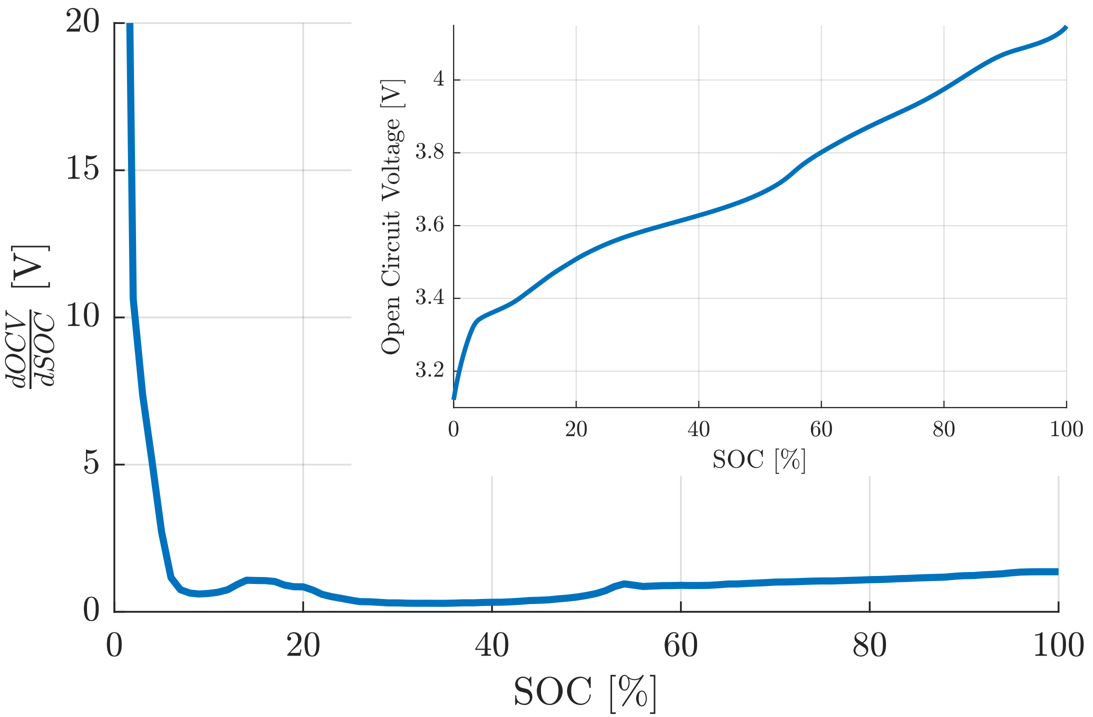
$$F_k = \frac{\partial f}{\partial x_k} = \begin{bmatrix} \frac{\partial z_{k+1}}{\partial z_k} & \frac{\partial z_{k+1}}{\partial v_{c,k}} \\ \frac{\partial v_{c,k+1}}{\partial z_k} & \frac{\partial v_{c,k+1}}{\partial v_{c,k}} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 - \frac{\Delta t_k}{R_1 C_1} \end{bmatrix}, \quad G_k = \frac{\partial f}{\partial w_k} = \begin{bmatrix} \frac{\partial z_{k+1}}{\partial w_{1,k}} & \frac{\partial z_{k+1}}{\partial w_{1,k}} \\ \frac{\partial v_{c,k+1}}{\partial w_{2,k}} & \frac{\partial v_{c,k+1}}{\partial w_{2,k}} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Measurement equation

$$h(x_k, u_k) = \mathrm{OCV}(z_k) + v_{c,k} + R_0 i_k \qquad H_k = \frac{\partial h}{\partial x_k} = \begin{bmatrix} \frac{\partial h}{\partial z_k} & \frac{\partial h}{\partial v_{c,k}} \end{bmatrix} = \begin{bmatrix} \frac{dOCV}{dz_k}(z_k) & 1 \end{bmatrix}$$

Estimating the OCV-Function Derivative

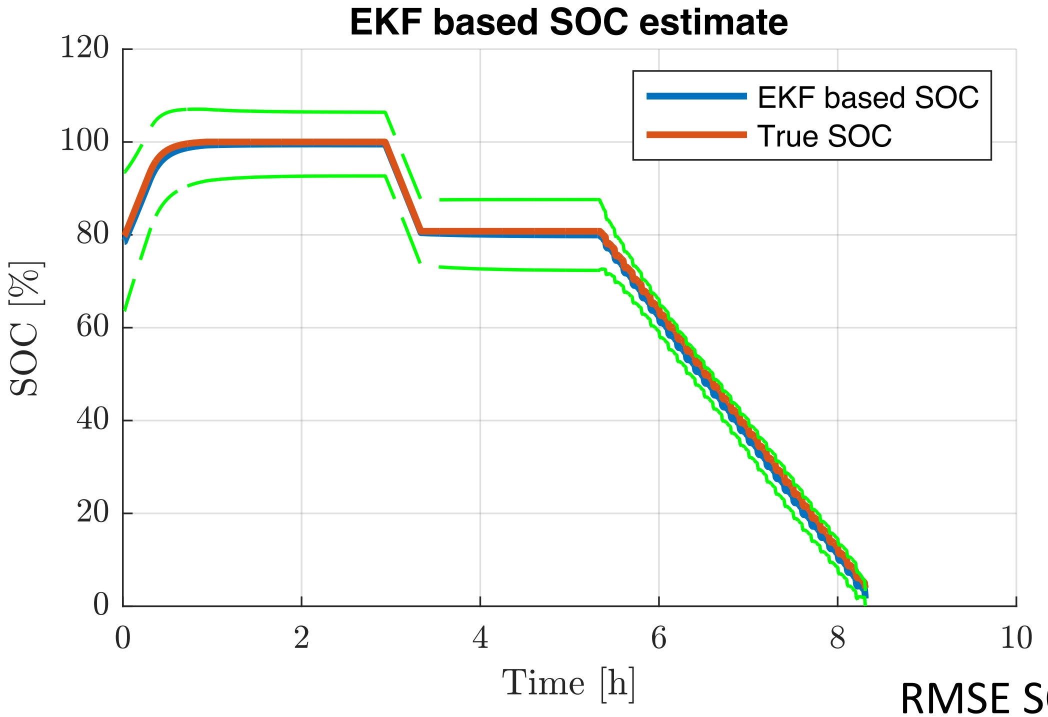
```
SOC = [0:0.01:1];                          % Select a SOC-grid
dOCVdSOC = gradient(OCVf(SOC), SOC);       % Numerical derivative
dOCVdz = @(z) interp1(SOC,dOCVdSOC,z);     % Interpolate
```

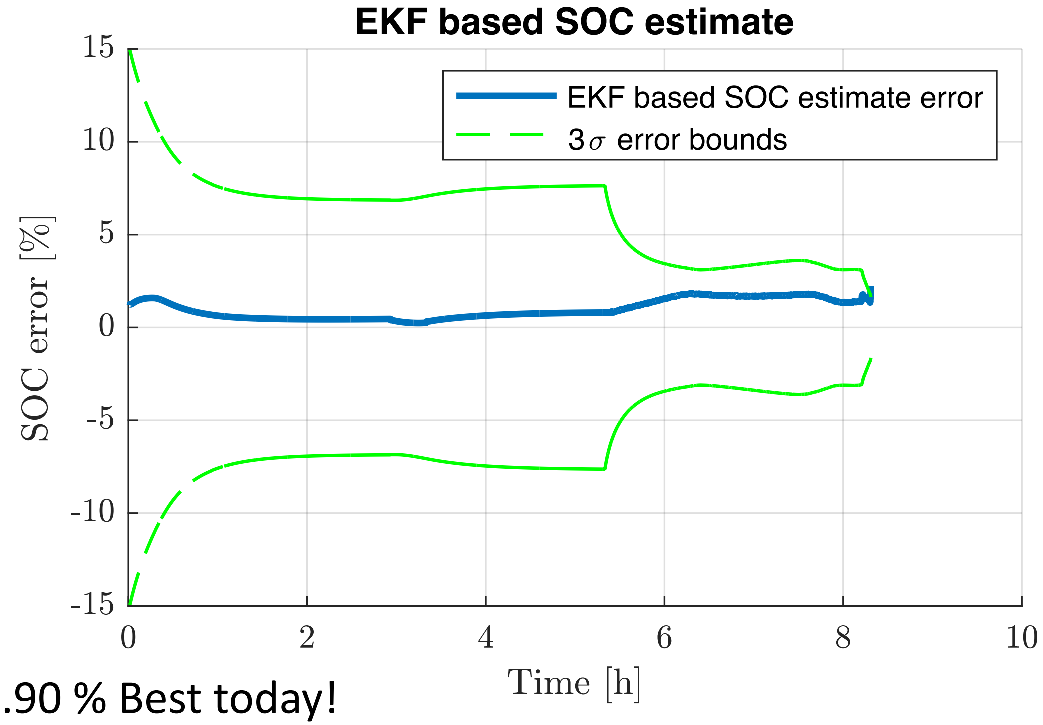


- Low derivative corresponds to little information in voltage measurement.
- The accuracy of the SOC estimate based on voltage is best at low and high SOCs.

EKF Results

- Variance of voltage measurement $R = (0.6)^2\ \mathrm{V}^2$ and process noise $Q = 1\ \mathrm{A}^2$
- Initial condition $z_{0|-1} = \mathrm{SOC}(v_0),\ v_{c,0|-1} = Ri_0,\ \ P_{0|-1} = \begin{bmatrix} 0.05^2 & 0 \\ 0 & R^2 \end{bmatrix}$





RMSE SOC: 0.90 % Best today!

Tuning EKF Filter

Initial condition $x_0 = \begin{bmatrix} z_0 \\ v_{c,0} \end{bmatrix}$

- Initial SOC could be estimated based on terminal voltage: $z_0 = \text{SOC}(v_0)$
- Capacitance voltage is less critical and can be approximated as $v_{c,0} = 0$

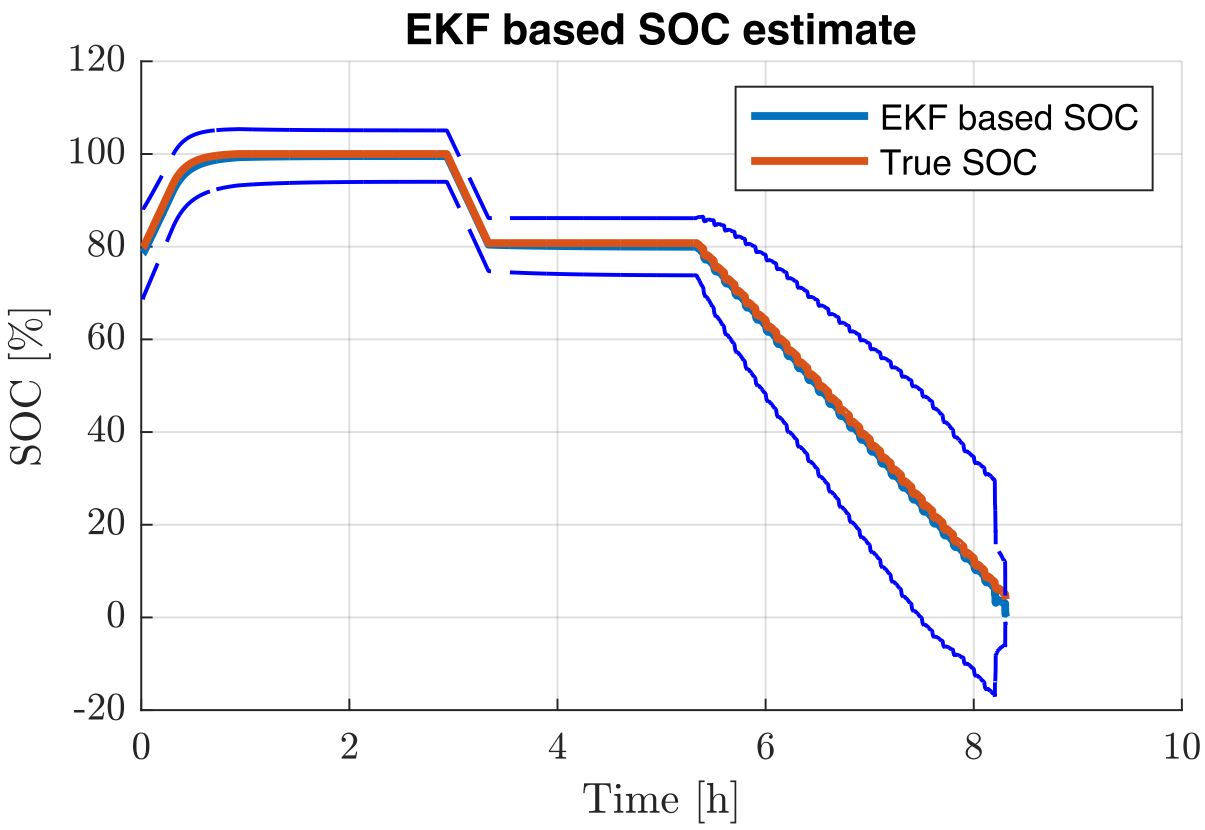
The standard deviations σ_x (~ uncertainties) could be seen as **tuning variables:**

Model uncertainty: $Q = \begin{bmatrix} \sigma_z^2 & 0 \\ 0 & \sigma_{v_c}^2 \end{bmatrix}$, Measurement uncertainty: $R = \sigma_v^2$

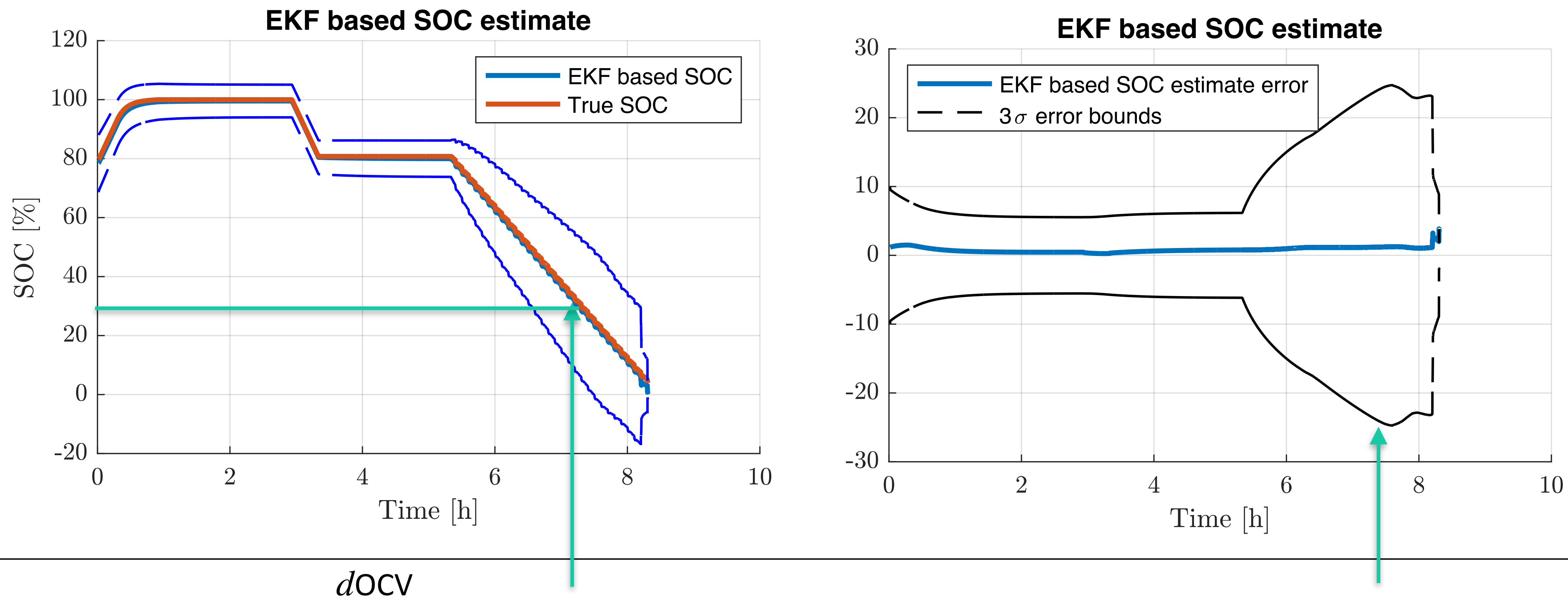
Initial condition uncertainty: $P_0 = \begin{bmatrix} \sigma_{z_0}^2 & 0 \\ 0 & \sigma_{v_{c,0}}^2 \end{bmatrix}$

- The initial guess should be of a reasonable order of magnitude. For example, if the initial SOC uncertainty is approximately ± 0.05, then $\sigma_{z_0} = 0.05$ is a good starting point.
- **Q/R Ratio Importance:** Balances trust between model predictions (Q) and measurements (R); higher Q/R favors measurements, lower Q/R favors model—crucial for filter stability and performance.

The blue interval shows $3\sigma_z$-confidence bands.
With a gaussian assumption $z \in [\hat{z} \pm 3\sigma_z]$ with 99.7%



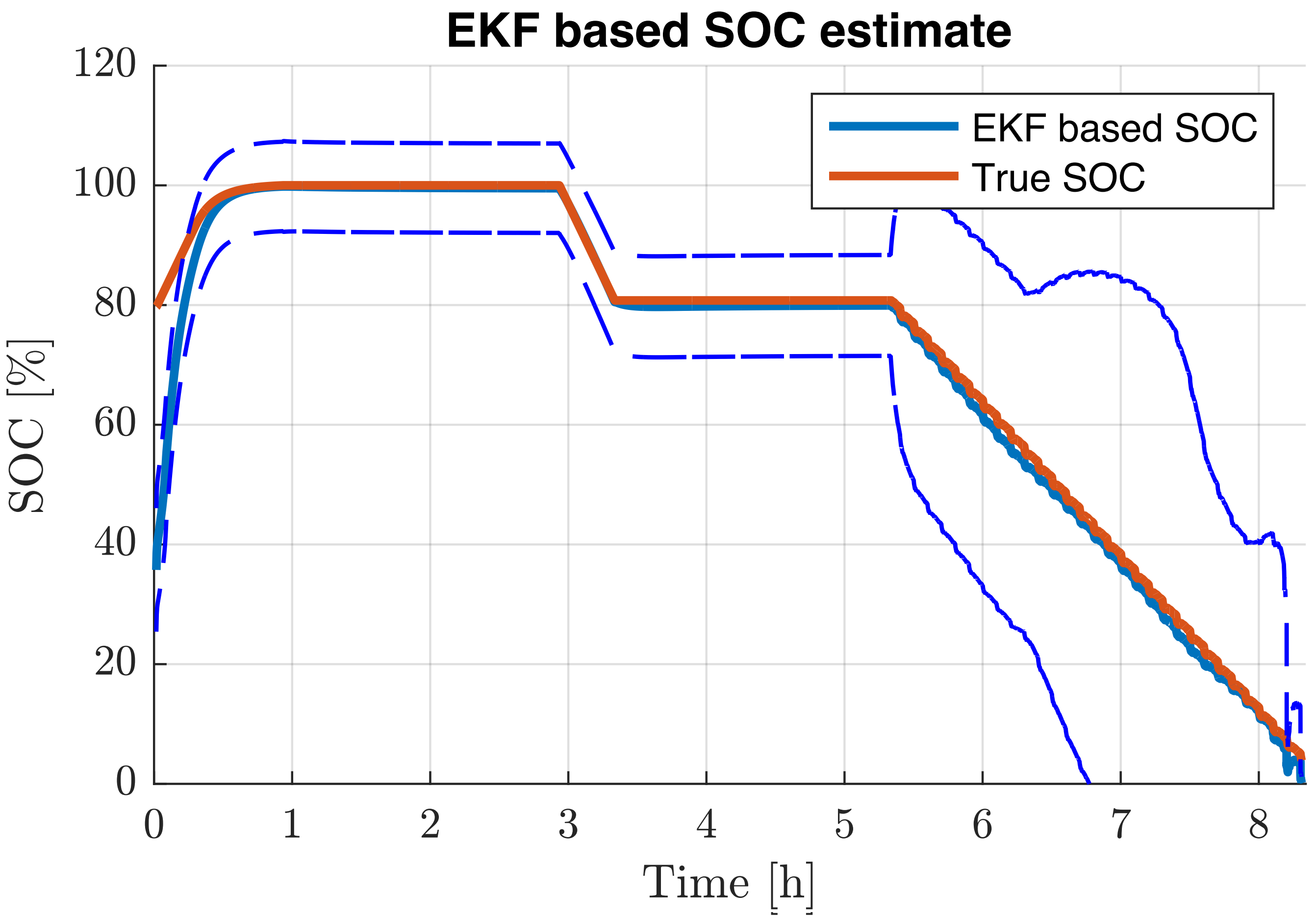
SOC Estimate and Error with Uncertainty Estimates



$\frac{d\mathrm{OCV}}{dz}$ has a minimum at 30 % SOC, which gives high SOC uncertainty at that point.

Tuning Examples

- What happens if the initial SOC is 50% off?
- SOC estimate is good after 3 hours.
- The uncertainty bounds are too narrow at the start!
    - Fix by increasing initial SOC uncertainty to about $\sigma_{z_0} = 0.5$
        - SOC estimate good from start!
    - Fix by trusting the measurement more, i.e., increasing Q/R by a factor of 10.
        - SOC estimate is good after 30 min
        - the confidence band is initially too narrow, and relying more on voltage-based SOC estimates will degrade the SOC performance at lower SOCs.



Learning Outcomes - SOC-estimation

By the end of this lecture, you should be able to:

- Implement SOC estimation using:
    - Coulomb counting
    - Voltage-based approaches
        - Terminal voltage methods
        - The Tino method
    - EKF-based methods with:
        - Thevenin equivalent model
        - R-RC model
- Understand the pros and cons of different SOC estimation methods, including EKF-based approaches.
- Understand how different uncertainty parameters (Q, R, initial P_0) affect SOC estimation performance, and apply this knowledge to effectively tune the Extended Kalman Filter (EKF).
- Understand how the derivative of the OCV function impacts SOC estimation uncertainty.

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