Prepare to Implement EKF for SOC-estimation

State equations:

$$\begin{cases} z_{k+1} = z_k + \dfrac{\Delta t_k}{Q} i_k + w_{1,k} \\ v_{c,k+1} = \left(1 - \dfrac{\Delta t_k}{R_1 C_1}\right) v_{c,k} + \dfrac{\Delta t_k}{C_1} i_k + w_{2,k} \end{cases}$$

Measurement equation:

$$v_k = \underbrace{ocv[z_k] + v_{c,k} + R_0 i_k}_{= h(x_k, u_k)} + e_k$$

Find: f, $F_k = \dfrac{\partial f}{\partial x}$, $G_k = \dfrac{\partial f}{\partial w}$, h, $H_k = \dfrac{\partial h}{\partial x}$, $\hat{x}_0$, P_0, Q, and R

Variables

$$x_k = \begin{bmatrix} z_k \\ v_{c,k} \end{bmatrix} \quad u_k = \begin{bmatrix} i_k \\ \Delta t_k \end{bmatrix} \quad y_k = v_k \quad w_k = \begin{bmatrix} w_{1,k} \\ w_{2,k} \end{bmatrix}$$

Covariances and initial guess

$$\Sigma_{w_k} = Q = \begin{bmatrix} \sigma_z^2 & 0 \\ 0 & \sigma_{v_c}^2 \end{bmatrix} \qquad \Sigma_{e_k} = R = \sigma_v^2$$

$$x_0 = \begin{bmatrix} z_0 \\ v_{c,0} \end{bmatrix} \qquad P_0 = \Sigma_{x_0} = \begin{bmatrix} \sigma_{z_0}^2 & 0 \\ 0 & \sigma_{v_{c,0}}^2 \end{bmatrix}$$

State equation:

$$\underbrace{\begin{bmatrix} z_{k+1} \\ v_{c,k+1} \end{bmatrix}}_{x_{k+1}} = f(x_k, u_k, w_k) = \begin{bmatrix} z_k + \dfrac{\Delta t_k}{Q} i_k + w_{1,k} \\ \left(1 - \dfrac{\Delta t_k}{R_1 C_1}\right) v_{c,k} + \dfrac{\Delta t_k}{C_1} \cdot i_k + w_{2,k} \end{bmatrix}$$

$$F_k = \frac{\partial f}{\partial x} = \begin{bmatrix} \frac{\partial z_{k+1}}{\partial z_k} & \frac{\partial z_{k+1}}{\partial v_{c,k}} \\ \frac{\partial v_{c,k+1}}{\partial z_k} & \frac{\partial v_{c,k+1}}{\partial v_{c,k}} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 - \frac{\Delta t_k}{R_1 C_1} \end{bmatrix}$$

$$G_k = \frac{\partial f}{\partial w} = \begin{bmatrix} \frac{\partial z_{k+1}}{\partial w_{1,k}} & \frac{\partial z_{k+1}}{\partial w_{2,k}} \\ \frac{\partial v_{c,k+1}}{\partial w_{1,k}} & \frac{\partial v_{c,k+1}}{\partial w_{2,k}} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Measurement equation

$$h(x_k, v_k) = ocv(z_k) + v_{c,k} + R_0 i_k$$

$$H_k = \frac{\partial h}{\partial x_k} = \begin{bmatrix} \frac{\partial h}{\partial z_k} & \frac{\partial h}{\partial v_{c,k}} \end{bmatrix} = \begin{bmatrix} \frac{\partial OCV}{\partial z_k}(z_k) & 1 \end{bmatrix}$$

↖ Prepared in lab code