Number theory, Talteori 6hp, Kurskod TATA54, Provkod TEN1
September 1, 2018
LINKÖPINGS UNIVERSITET
Matematiska Institutionen
Examinator: Jan Snellman

All problems are worth 3 points. To receive full points, a solution needs to be complete. Indicate which theorems from the textbook that you have used, and include all auxillary calculations.

No aids, no calculators, tables, nor textbooks.

8-10p: grade 3, 11-13p: grade 4, 14-18p grade 5.

1) Find all solutions to

$$\begin{aligned} 2x - 18 &\equiv 20 \mod 4 \\ 3x - 3 &\equiv 30 \mod 9 \end{aligned}$$

2) For which primes p is the congruence

$$x^2 \equiv 7 \mod p$$

solvable?

3) Find the continued fraction expansion of $13/29$.

4) Define the numerical function s by

$$s(n) = \sum_{d|n} \mu(d)d.$$

Show that s is multiplicative, and find an explicit formula (possibly involving the factorization of the argument) for s. Determine

$$\sum_{d|n} s(d)\phi(n/d).$$

5) Consider the Diophantine equation $x^2 - 5y^2 = 1$. Find the smallest solution, i.e. the integer solution (x, y) with $x^2 + y^2$ minimal. Determine all solutions.

6) Find all solutions to

$$x^2 + x + 1 \equiv 0 \mod 27.$$