Control of Autonomous Vehicles II

TSFS12: Autonomous Vehicles –planning, control, and learning systems

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Closed Loop Rapidly-Exploring Random Tree (CL-RRT)

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Today I will present the method Closed Loop Rapidly-Exploring Random Tree. The presentation will be based on the following material from previous lectures:

- Kinematic Model (lecture 3)
- Dubins Car (lecture 3)
- Rapidly-exploring random tree (lecture 4)
- Pure-Pursuit Control (lecture 6)

The main reference is the paper:
Real-Time Motion Planning With Applications to Autonomous Urban Driving, Kuwata et.al.,IEEE TRANSACTIONS ON CONTROL SYSTEMS TECHNOLOGY, VOL. 17, NO. 5, SEPTEMBER 2009

Material from Previous Lectures

Kinematic Model (Lecture 3)

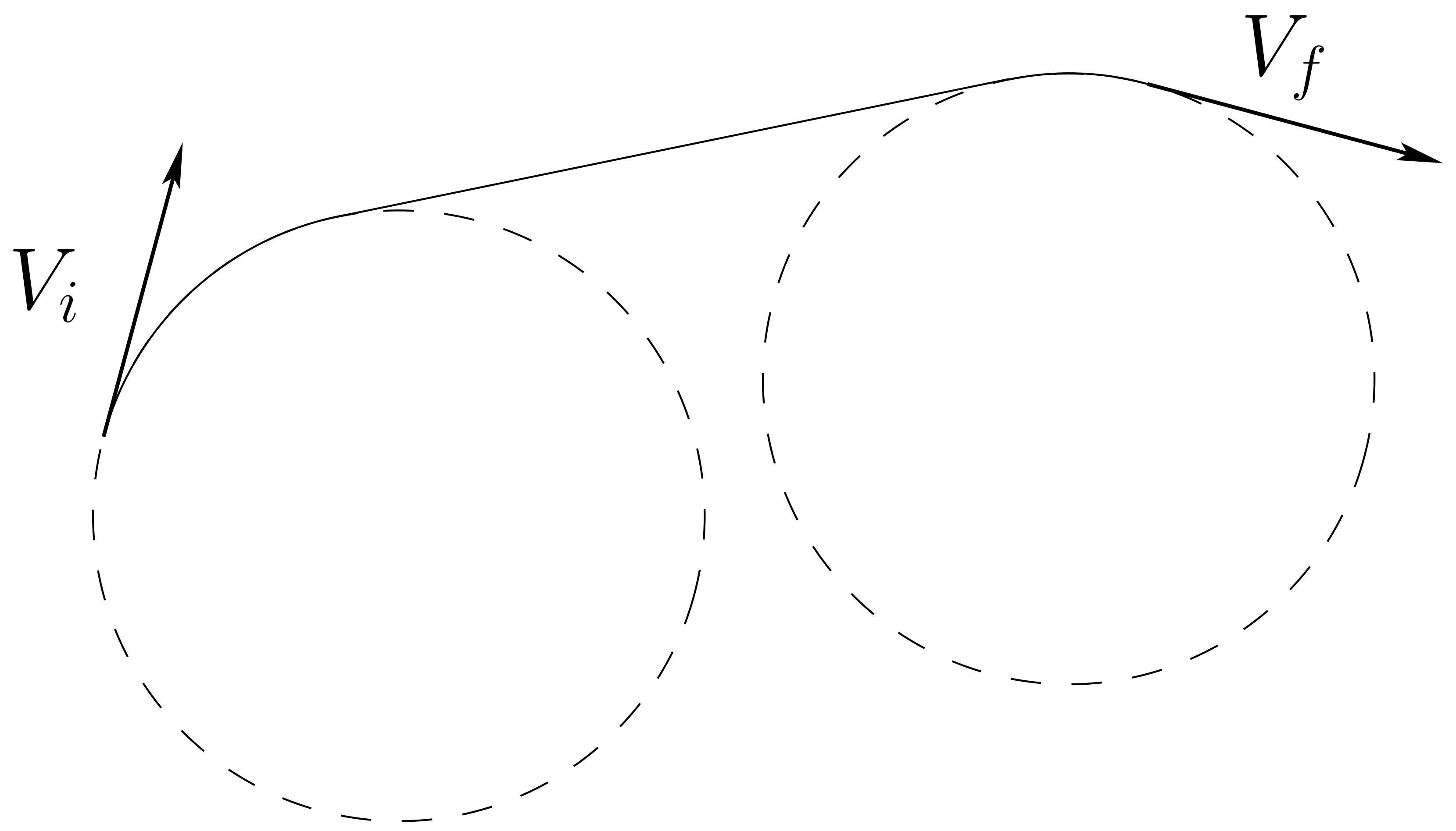
Example of a kinematic model:

$$
\begin{aligned}
\dot{x} &= v\cos\theta \\
\dot{y} &= v\sin\theta \\
\dot{\theta} &= v\tan\delta/l \\
\dot{\delta} &= u_1 \\
\dot{v} &= a \\
\dot{a} &= u_2 \\
\delta &\in [-\delta_{max}, \delta_{max}] \\
u_1 &\in [-\dot{\delta}_{max}, \dot{\delta}_{max}]
\end{aligned}
$$

Note that the steering angle and acceleration are states.
This gives a smoother trajectory.

Dubins Car (Lecture 3)

Problem: Find the shortest path between two points with the orientation specified at the initial and final point, and the turning radius limited from below

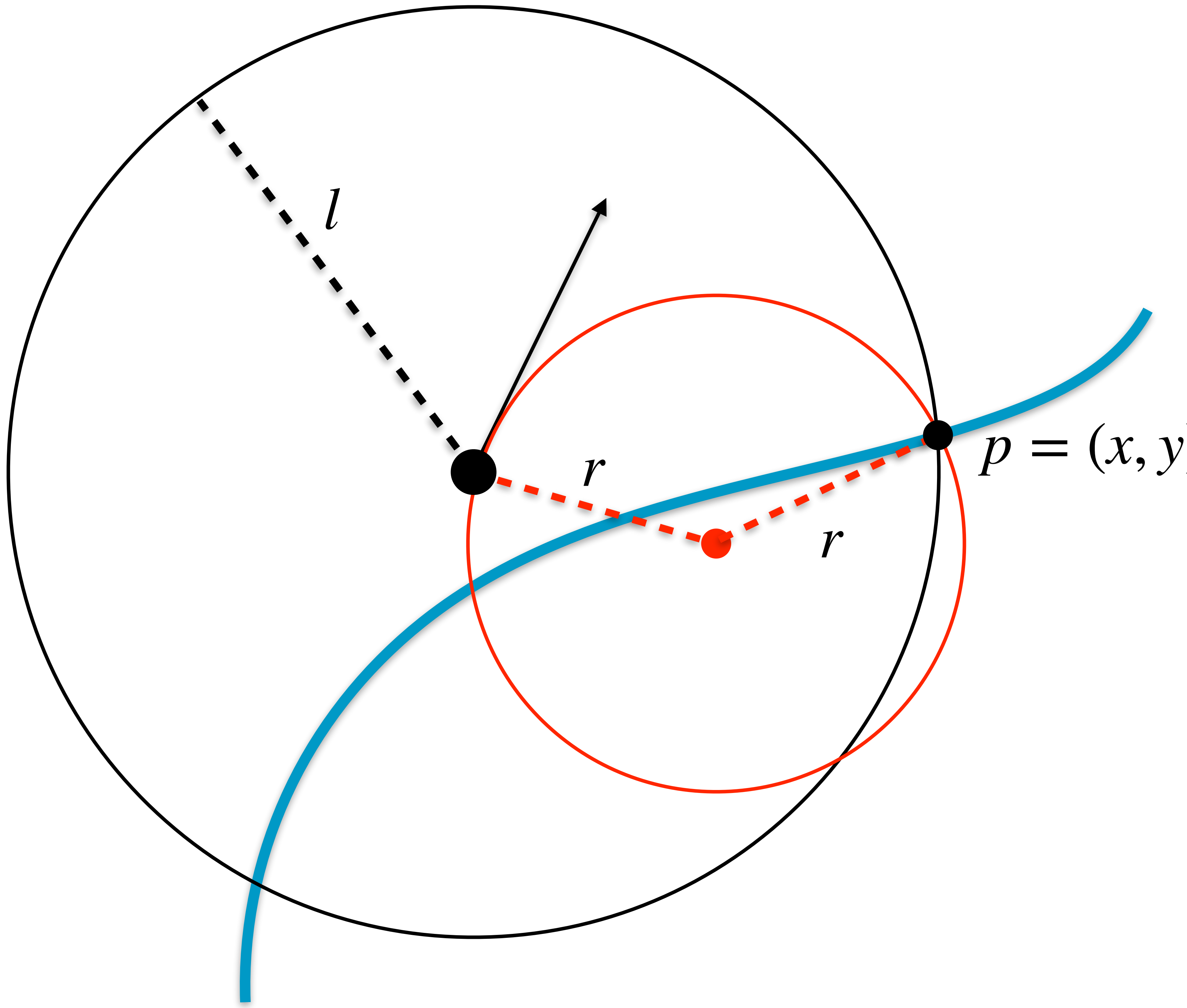


Solution: The optimal solution consists of segments with minimal radius R_{min} and straight lines.

Pure-pursuit control (Lecture 6)

- A simple control technique to compute the arc needed for a robot to get back on path
- With a look-ahead horizon, l, find point $p = (x, y)$ on the path to aim for
- Compute the turning radius r to get there
- For a single-track robot, this corresponds to a steering angle

$$\frac{1}{L}\tan\delta = \frac{1}{r}$$



Basic Version of RRT (Lecture 4)

Algorithm 1: RRT w/o. Differential Constraints

$\mathcal{V} \leftarrow \{q_I\}, \quad \mathcal{E} \leftarrow \emptyset;$
for $i = 1, \ldots, N$ **do**:
    $q_{\text{rand}} \leftarrow \text{Sample};$
    $q_{\text{nearest}} \leftarrow \text{Nearest}(\mathcal{G} = (\mathcal{V}, \mathcal{E}), q_{\text{rand}});$
    $q_{\text{new}} \leftarrow \text{Steer}(q_{\text{nearest}}, q_{\text{rand}});$
    if $\text{ObstacleFree}(q_{\text{nearest}}, q_{\text{new}})$ **then**
        $\mathcal{V} \leftarrow \mathcal{V} \cup \{q_{\text{new}}\};$
        $\mathcal{E} \leftarrow \mathcal{E} \cup \{(q_{\text{nearest}}, q_{\text{new}})\};$
return $\mathcal{G} = (\mathcal{V}, \mathcal{E});$

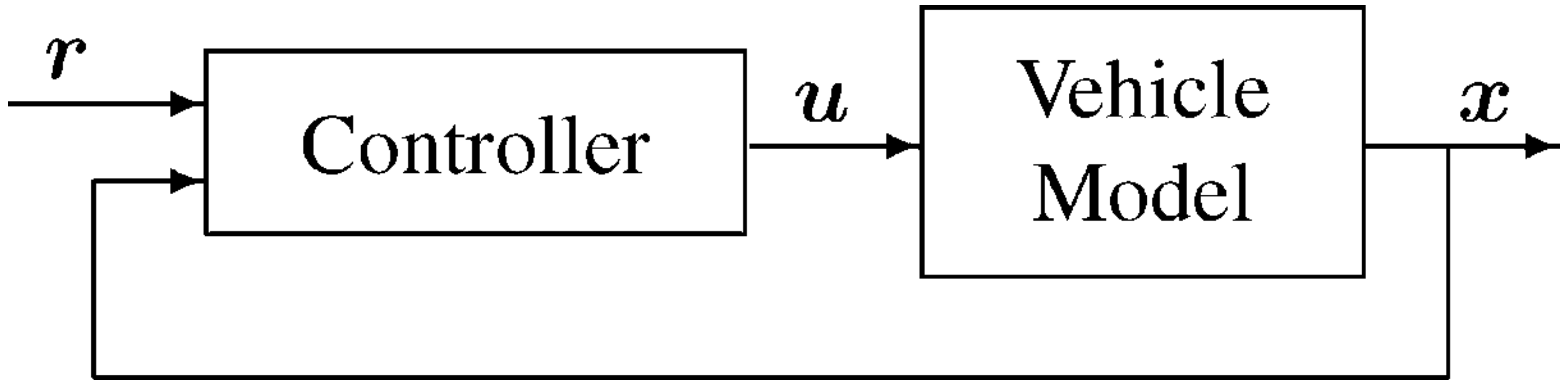
LaValle, S. M., & J. J. Kuffner Jr.: "Randomized kinodynamic planning”. *The International Journal of Robotics Research,* 20(5), 378-400, 2001.

Basic Version of RRT (Lecture 4)

- **Sample**: Gives a sample in the free state space.
- **Nearest**: Provides the vertex in the tree that is closest to the sampled state.
- **Steer**: In general this is a so called two-point boundary value problem (TPBV). Construct a path from the nearest vertex towards the sampled state, often with a maximum path length (alternative strategies exist).
- **ObstacleFree**: Checks whether the path from the closest vertex in the graph to the new state is collision free.

Closed Loop Rapidly-Exploring Random Tree (CL-RRT)

CL-RRT samples an input to the stable closed-loop system consisting of the vehicle and the controller.



- CL-RRT works for vehicles with unstable dynamics, such as cars and helicopters, by using a stabilizing controller.
- A single input to the closed-loop system can create a long trajectory (on the order of several seconds) while the controller provides a high-rate stabilizing feedback

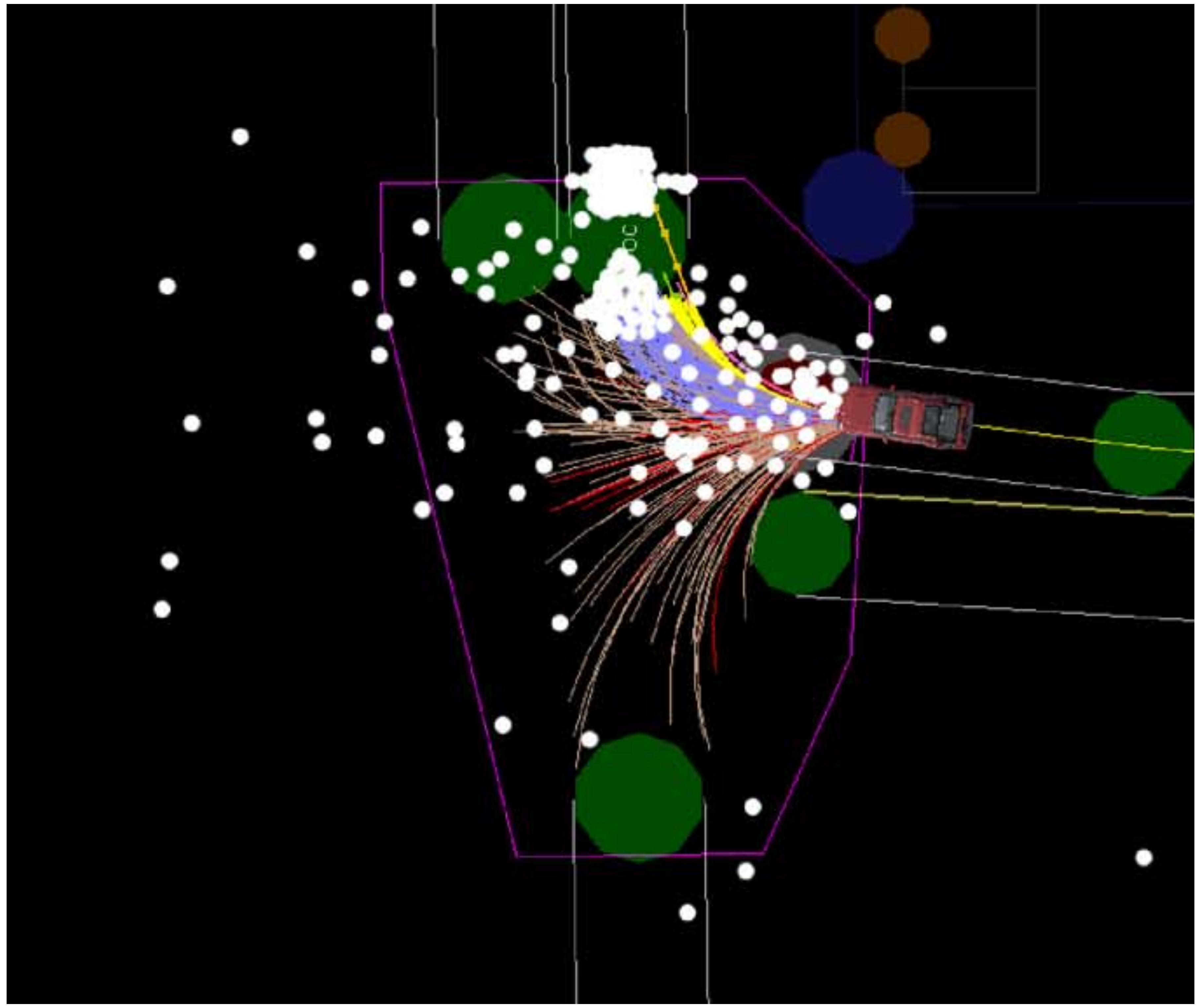
CL-RRT: Sampling strategies

Given a reference position and heading (x_0, y_0, θ_0) a sample point (s_x, s_y) is generated by:

$$\begin{bmatrix} s_x \\ s_y \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + r \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \end{bmatrix} \text{ with } \begin{cases} r = \sigma_r |n_r| + r_0 \\ \theta = \sigma_\theta n_\theta + \theta_0 \end{cases}$$

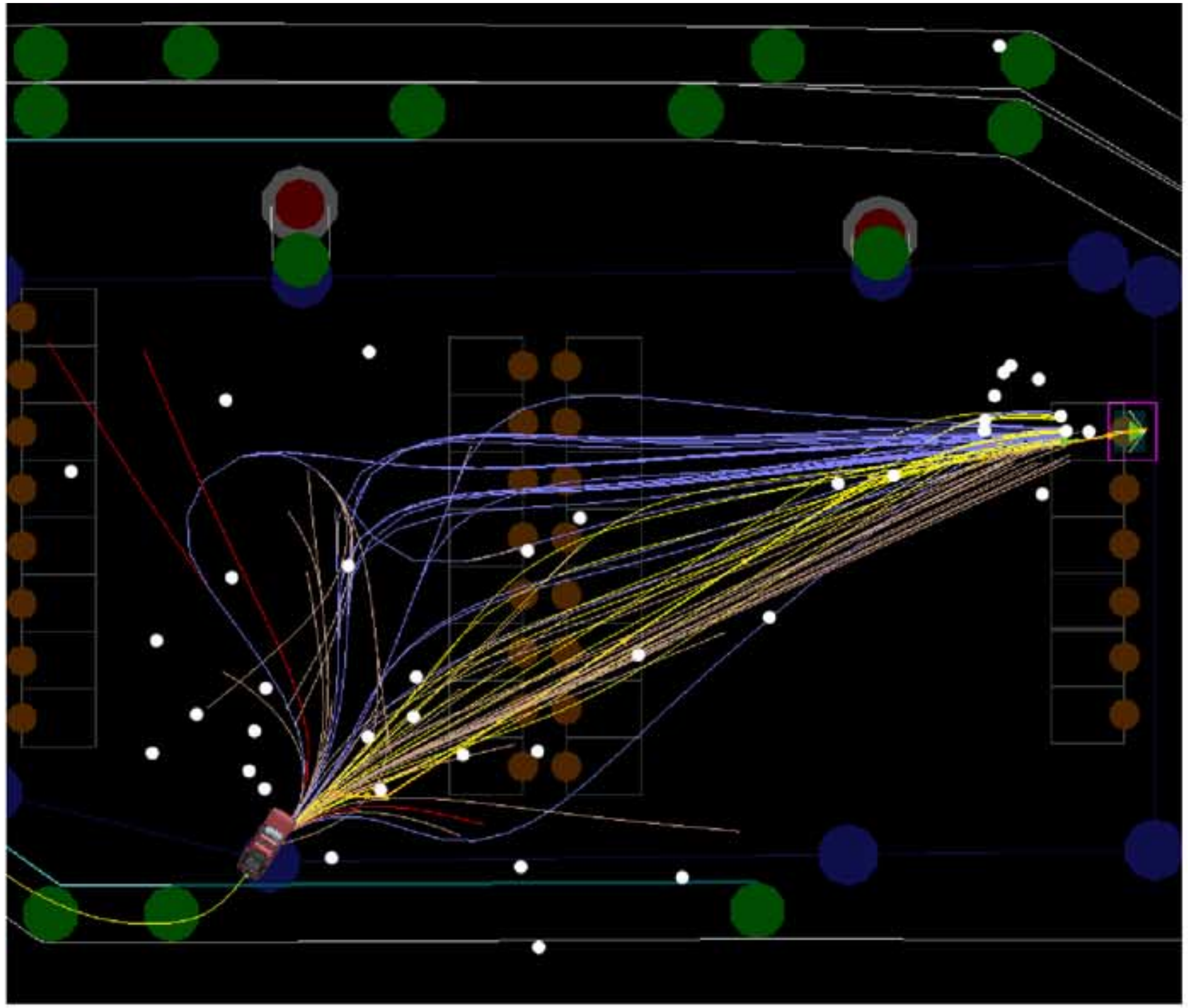
where n_r and n_θ are random variables with standard Gaussian distribution, σ_r is the standard deviation in the radial direction σ_θ is the standard deviation in the circumferential directions, and r_0 is an offset with respect to (x_0, y_0).

Example: Sampling at a right hand turn at an intersection



A wide and relatively short Gaussian distribution is used that covers the open space inside the intersection boundary. The value of σ_r is set to the distance to the goal and σ_θ is set at 0.4π.

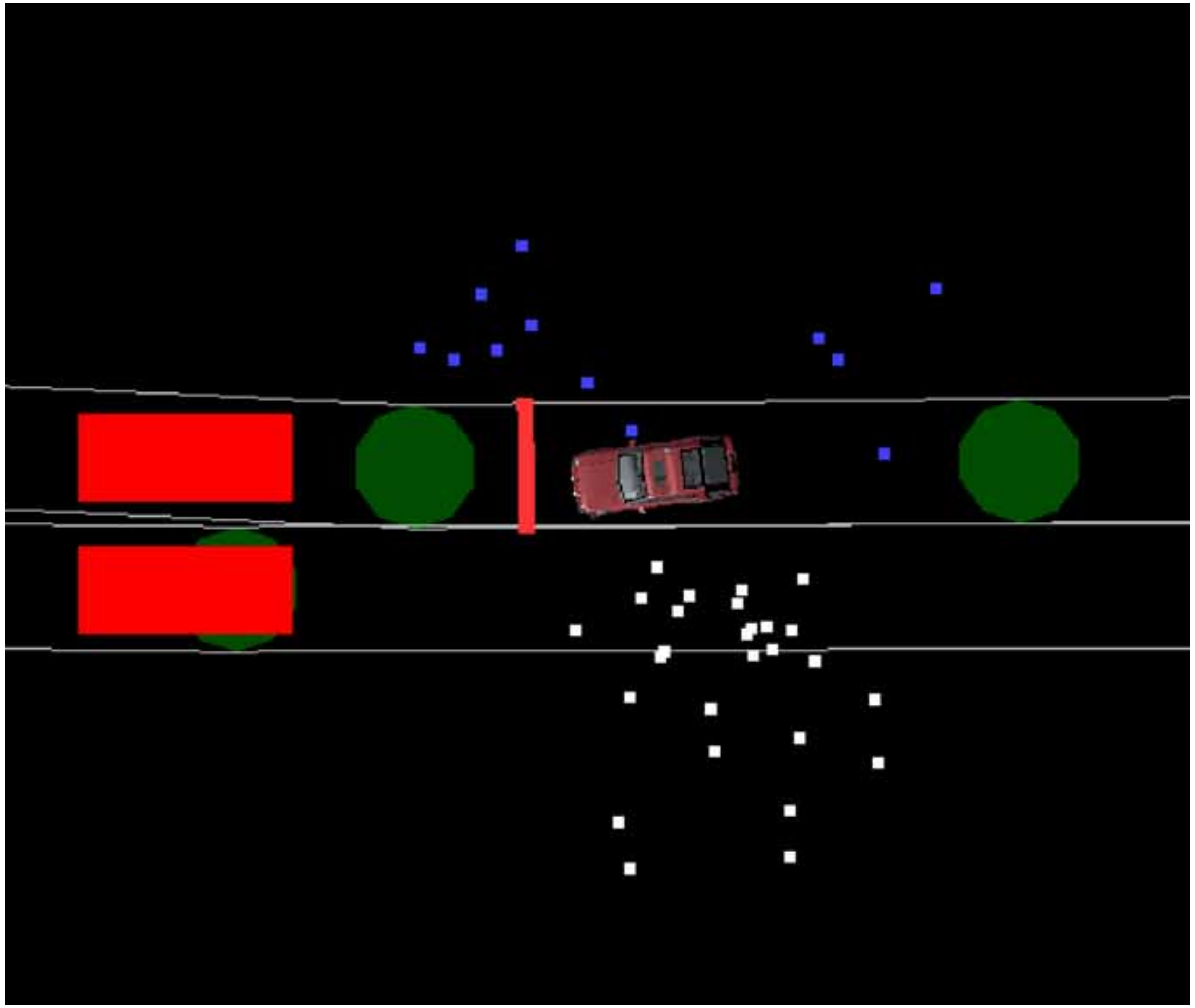
Example: Sampling at a parking lot



Sampling is taken both around the vehicle and around the parking spot.

Around the vehicle a wide and long distribution is used

Example: Sampling strategy for an U-turn

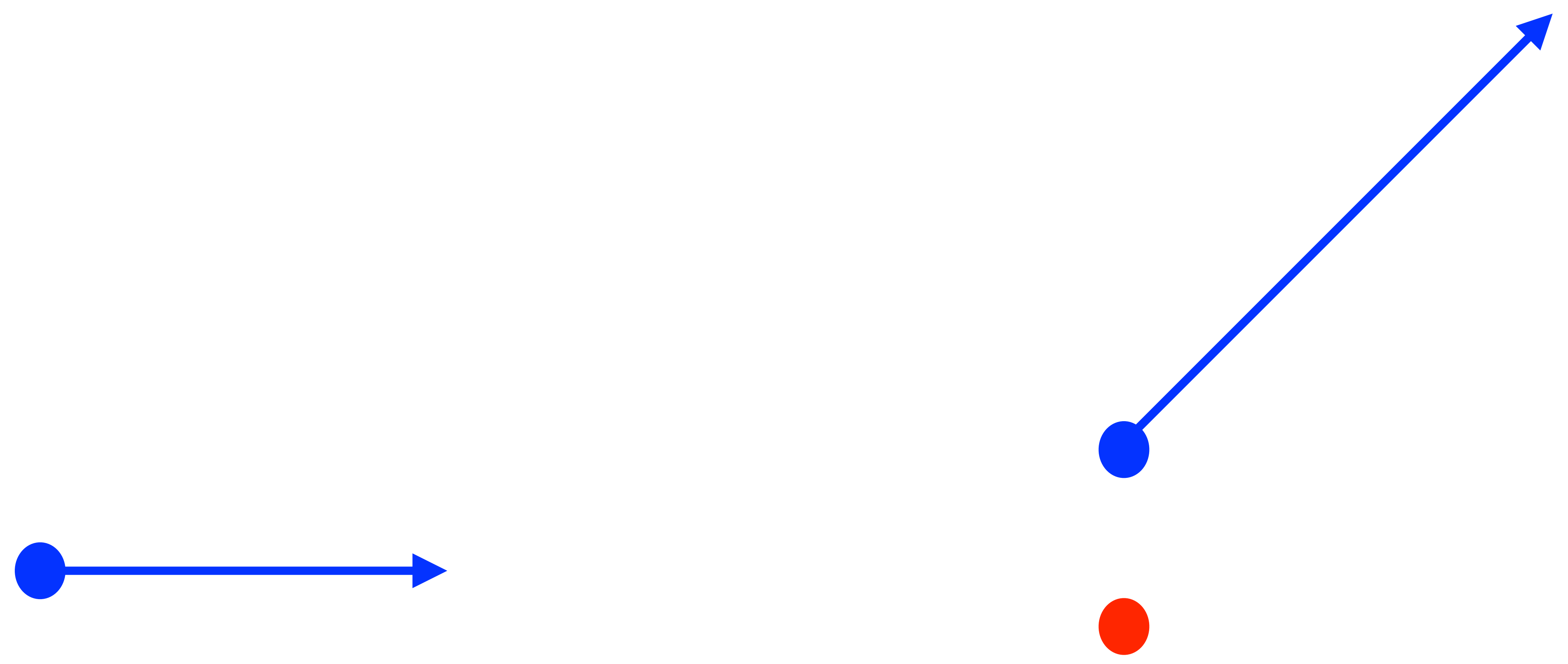


The vehicle is facing the road blockage (red).

Blue and white dots are reverse and forward manoeuvres, respectively.

CL-RRT: Heuristic

Which node (blue) is "closest" to the sample state (red)?

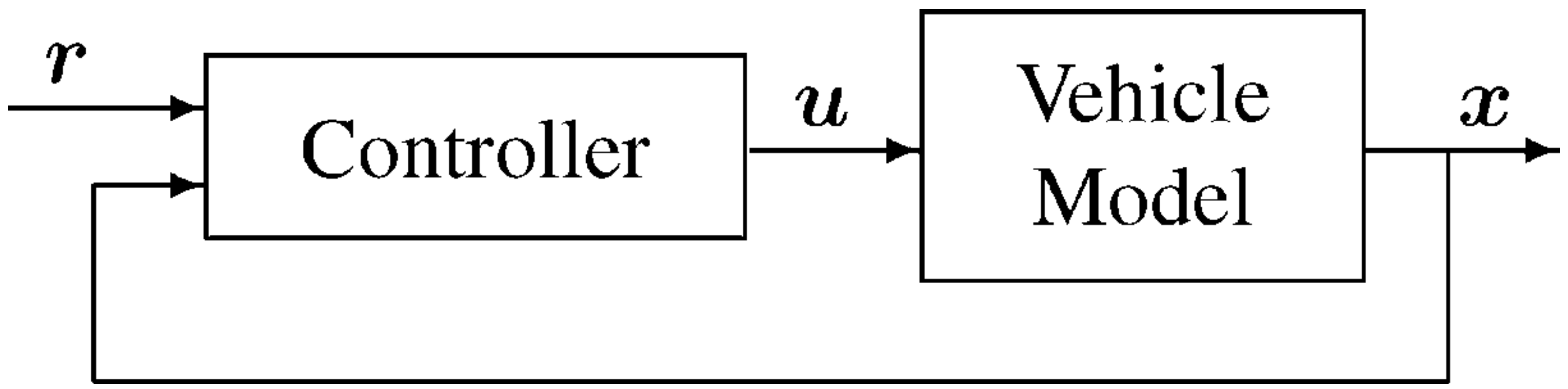


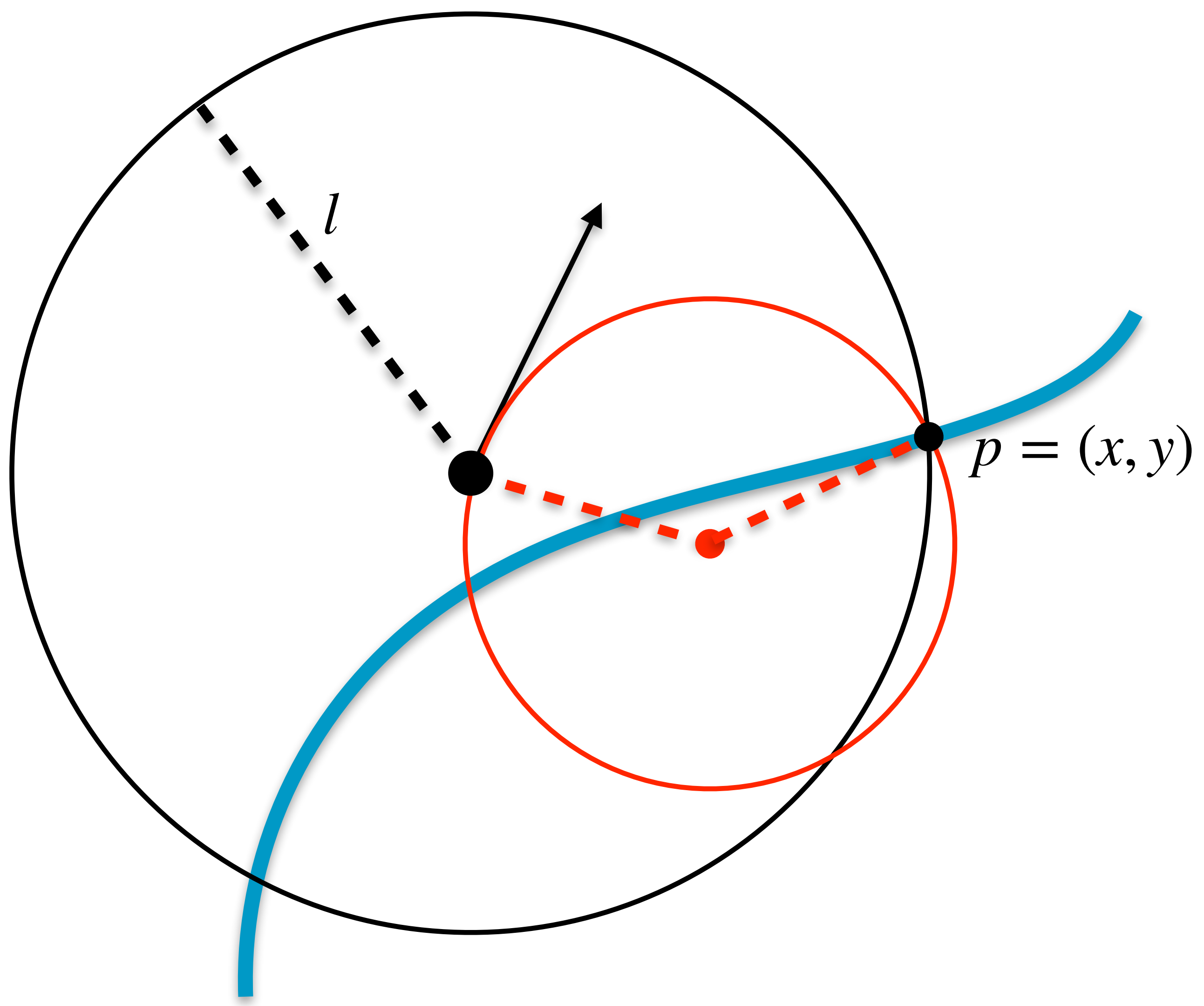
In RRT the distance was used, and the answer in this case the nearest node would be the one to the right.

In CL-RRT the Dubins distance is used, defined as the shortest path a with a minimal turning radius R_{min}. The nearest node would be the one to the left if R_{min} is sufficiently large. This is called the exploration heuristic.

CL-RRT: The controller

A pure-pursuit controller is used in the CL-RRT



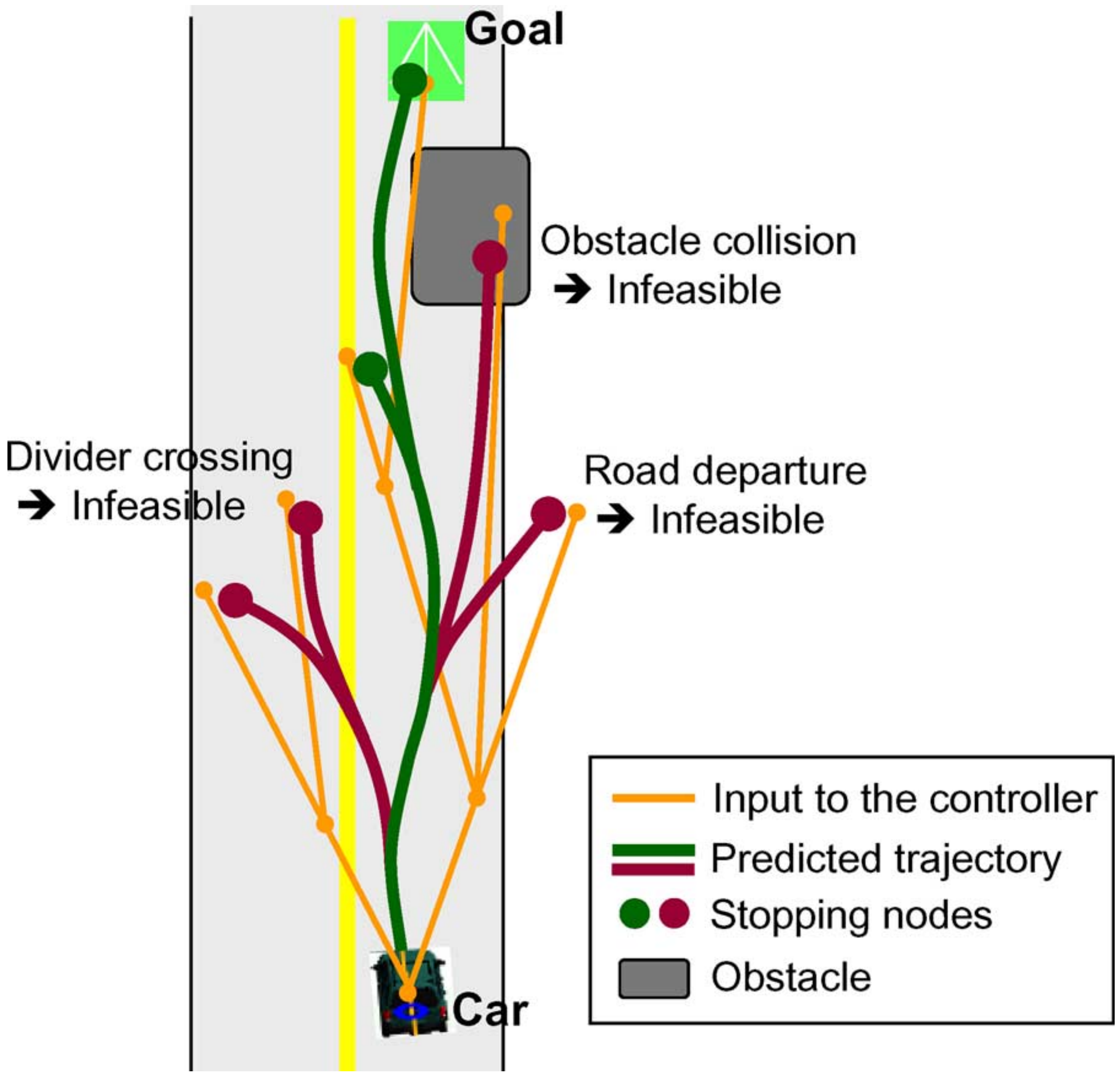


The next step is to describe how to construct reference paths for this controller.

CL-RRT: Expand the tree

Procedure to expand the (orange) tree

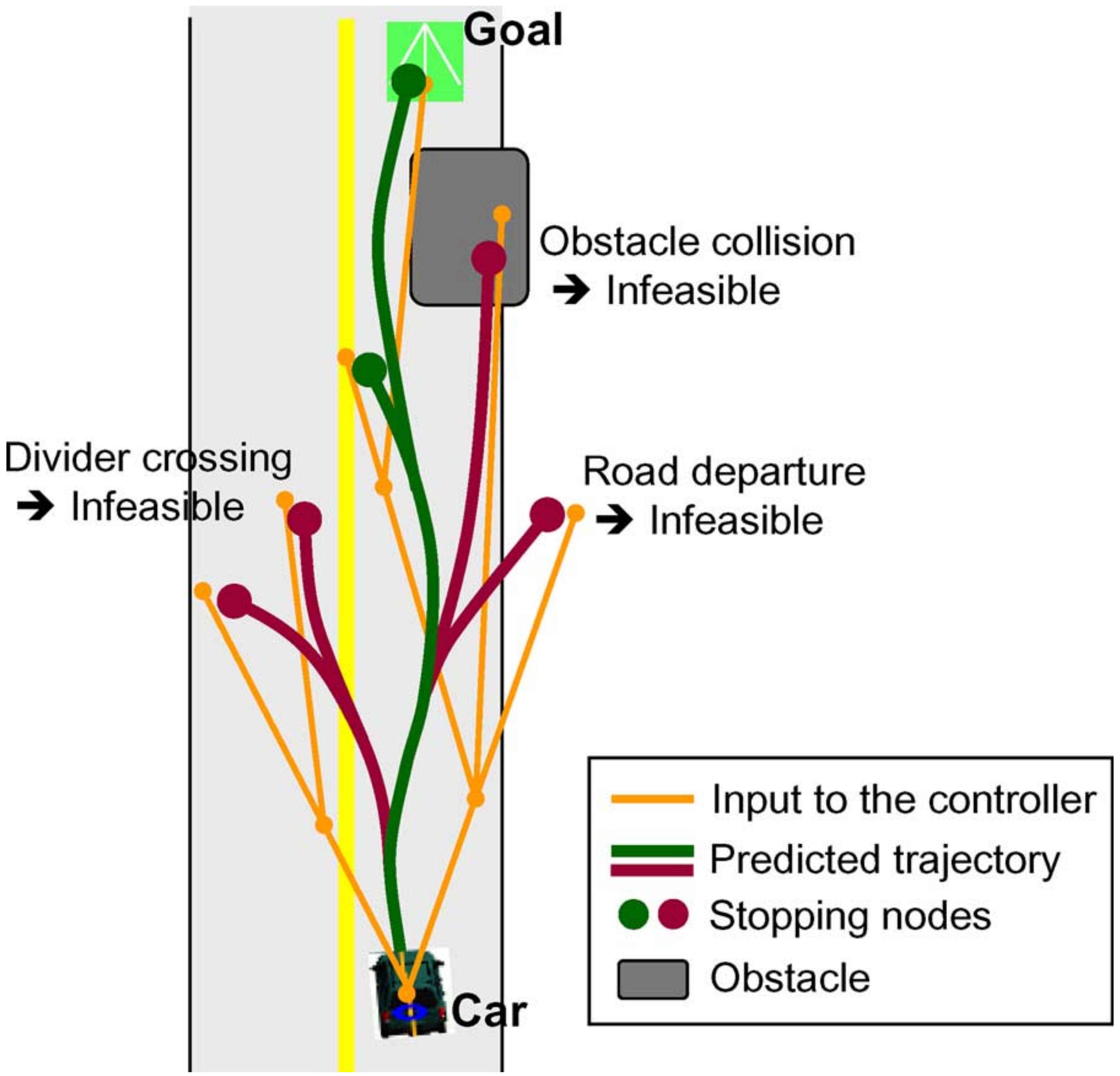
- Generate a sample position s.
- For each node q in the tree, in the order sorted by the Dubins distance $L_\rho(s)$, use the line segment between node q and sample s to extend the orange tree.
- Use the new reference path in the orange tree as input to the controller to generate a trajectory $\mathbf{x}(t)$, $t \in [t_1, t_2]$ until it stops (red and green curves).



CL-RRT: Expand the tree

- If $\mathbf{x}(t) \in \mathscr{X}_{free}(t)$ for all $t \in [t_1, t_2]$, then add the sample s to the tree and also some intermediate nodes.
- Else if the intermediate nodes are feasible, add them to the tree.

If no nodes were added above, then repeat the process with a new node q.

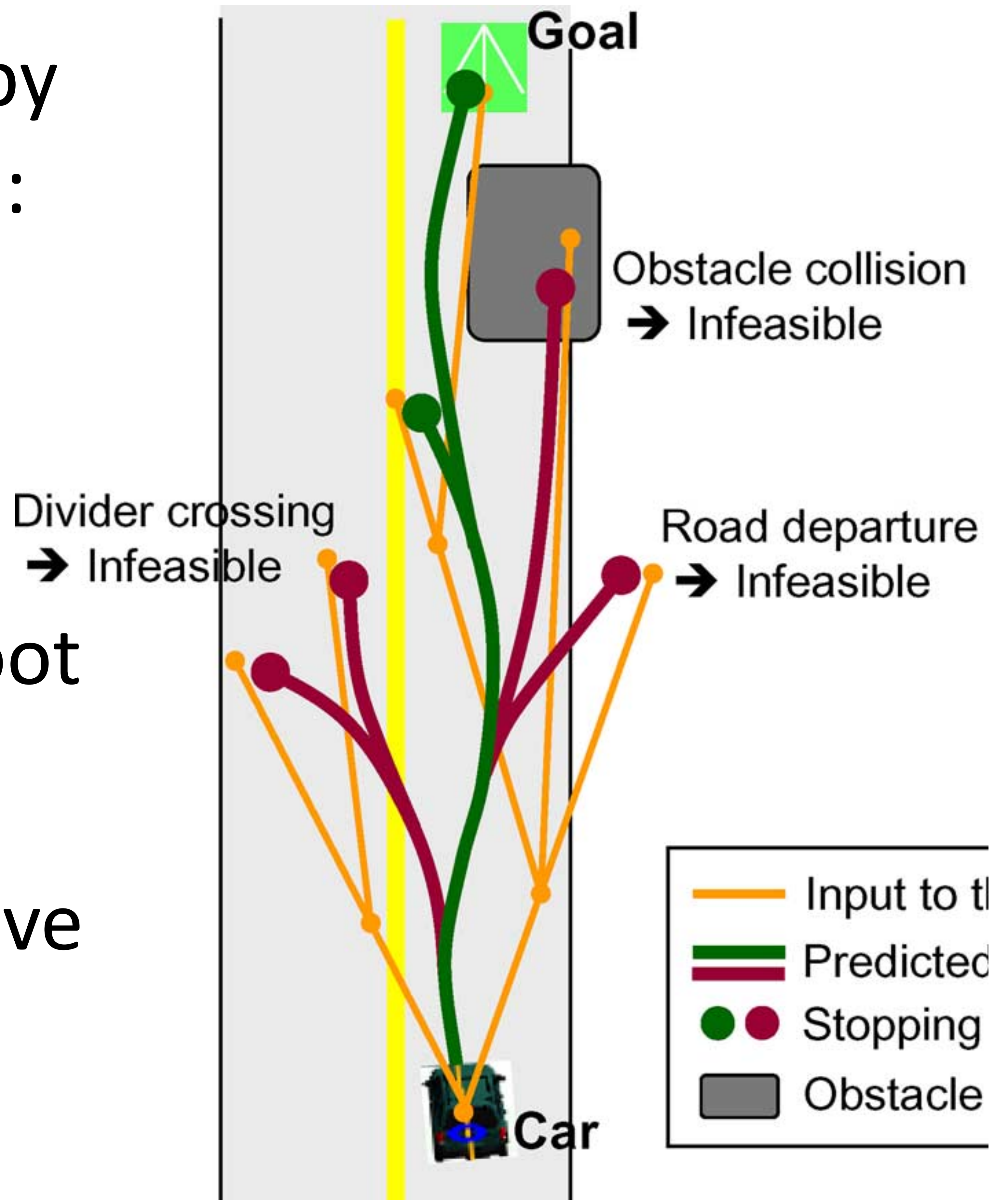


CL-RRT

The tree is expanded until the goal has been reached. After that the nodes are mainly sorted by ascending order of total cost to reach the sample:

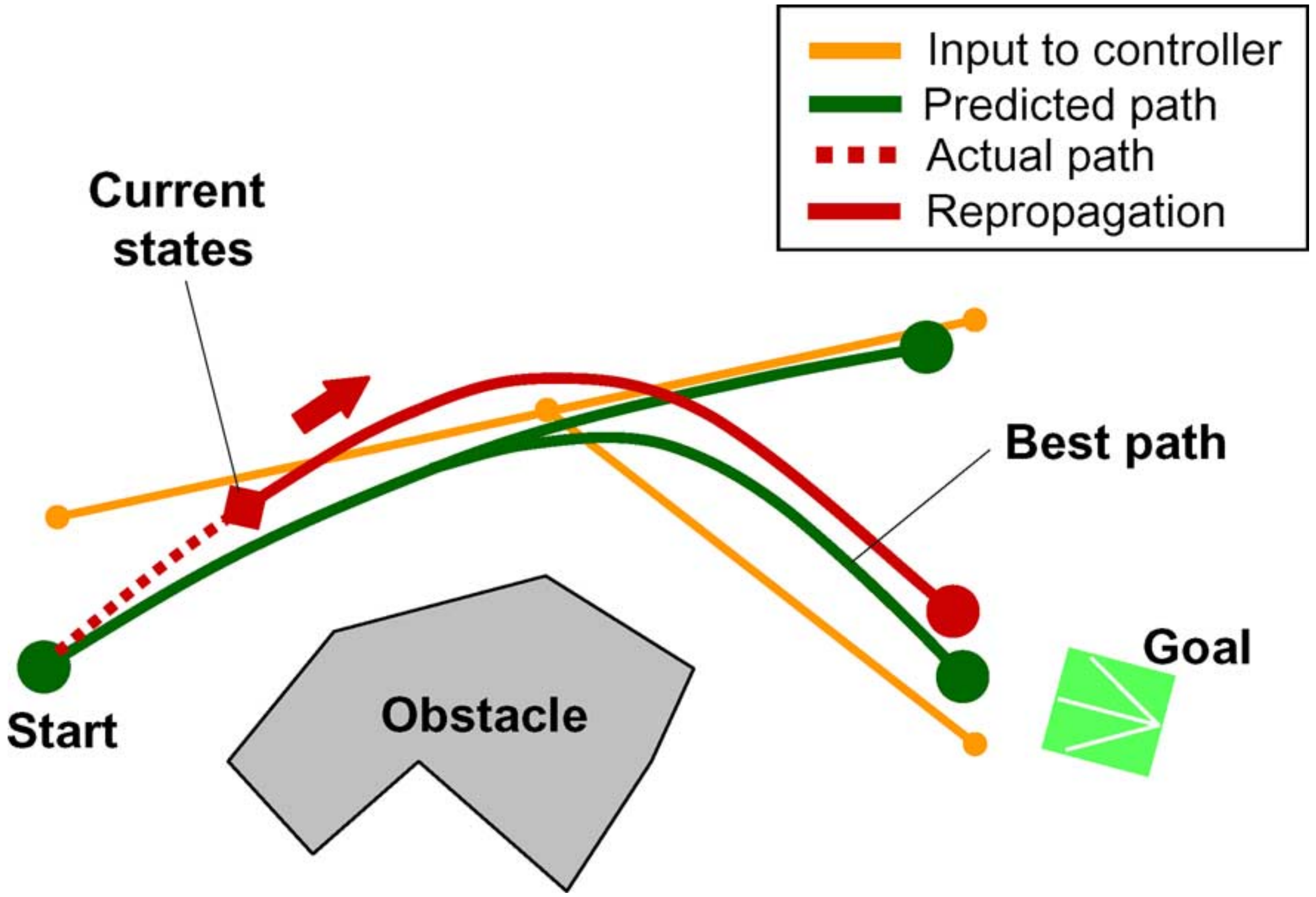
$$C_{total} = C_{cum}(q) + L_\rho(s)/v$$

where $C_{cum}(q)$ is the cumulative cost from the root of the tree to a node q, $L_\rho(s)$ is the Dubins distance, and v is the sampled speed. The objective is to make the new trajectories approach the shortest path. This is called the optimisation heuristic.



Replanning

When the vehicle has moved forward a step, the expansion of the tree continues and obsolete parts are removed.



See the reference on the first slide for further details

Artificial Potential Fields

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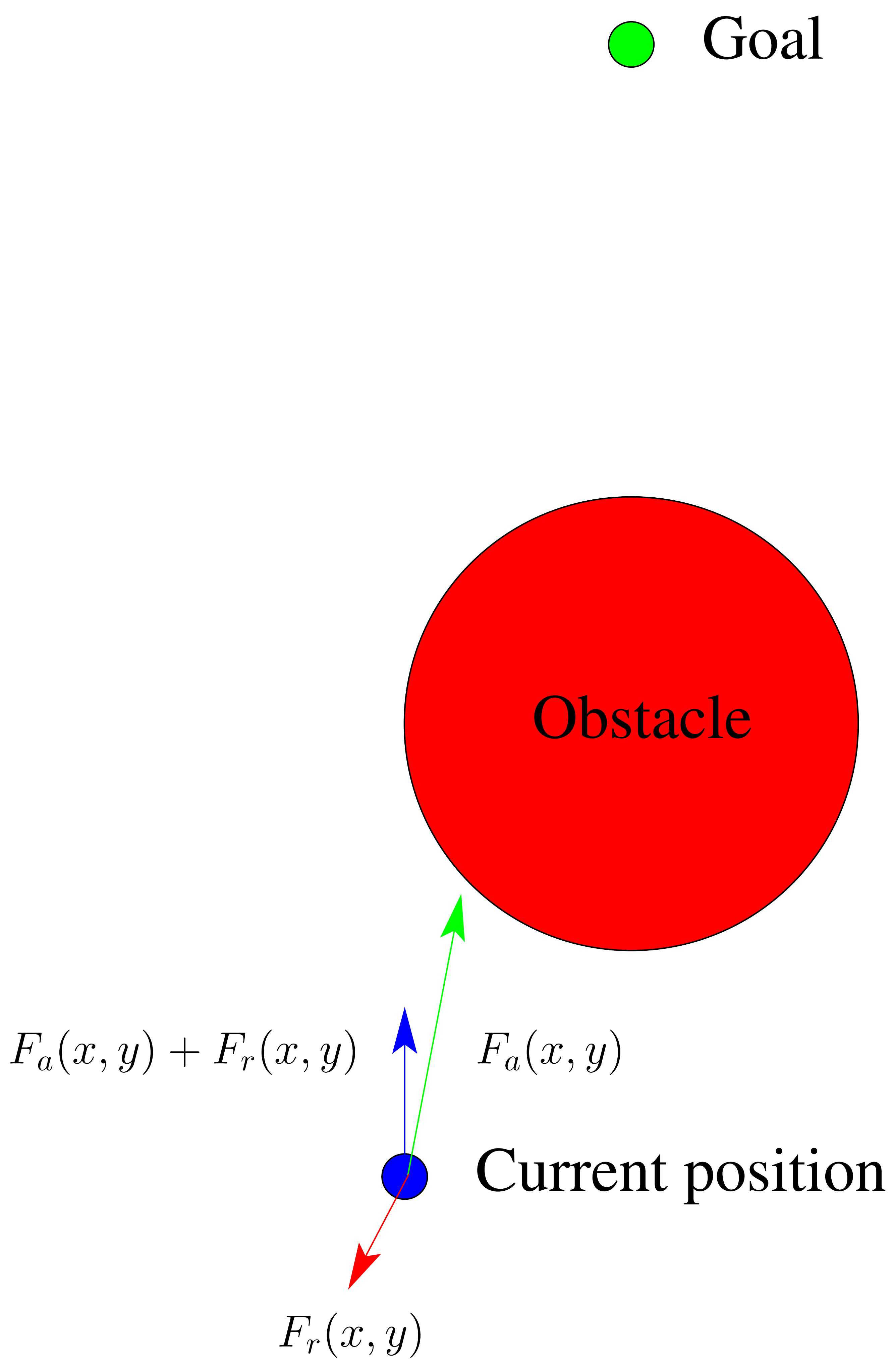
Artificial Potential Field Method

The example to the right will be used to illustrate the method. The objective for the agent (blue dot) is to reach the goal (green dot) avoiding the obstacle (red circle).

The $F_a(x, y)$idea is to construct two vector fields. The first vector field is called the attractive force and will push the agent towards the goal. The second vector field $F_r(x, y)$ is called the repulsive force and will push the agent away from the obstacle. The two vector fields are added and the sum

$$F(x, y) = F_a(x, y) + F_r(x, y)$$

is used to control the agent, e.g, by using a single integrator $\dot{\mathbf{q}}(x, y) = F(x, y)$ or a double integrator $\ddot{\mathbf{q}}(x, y) = F(x, y)$



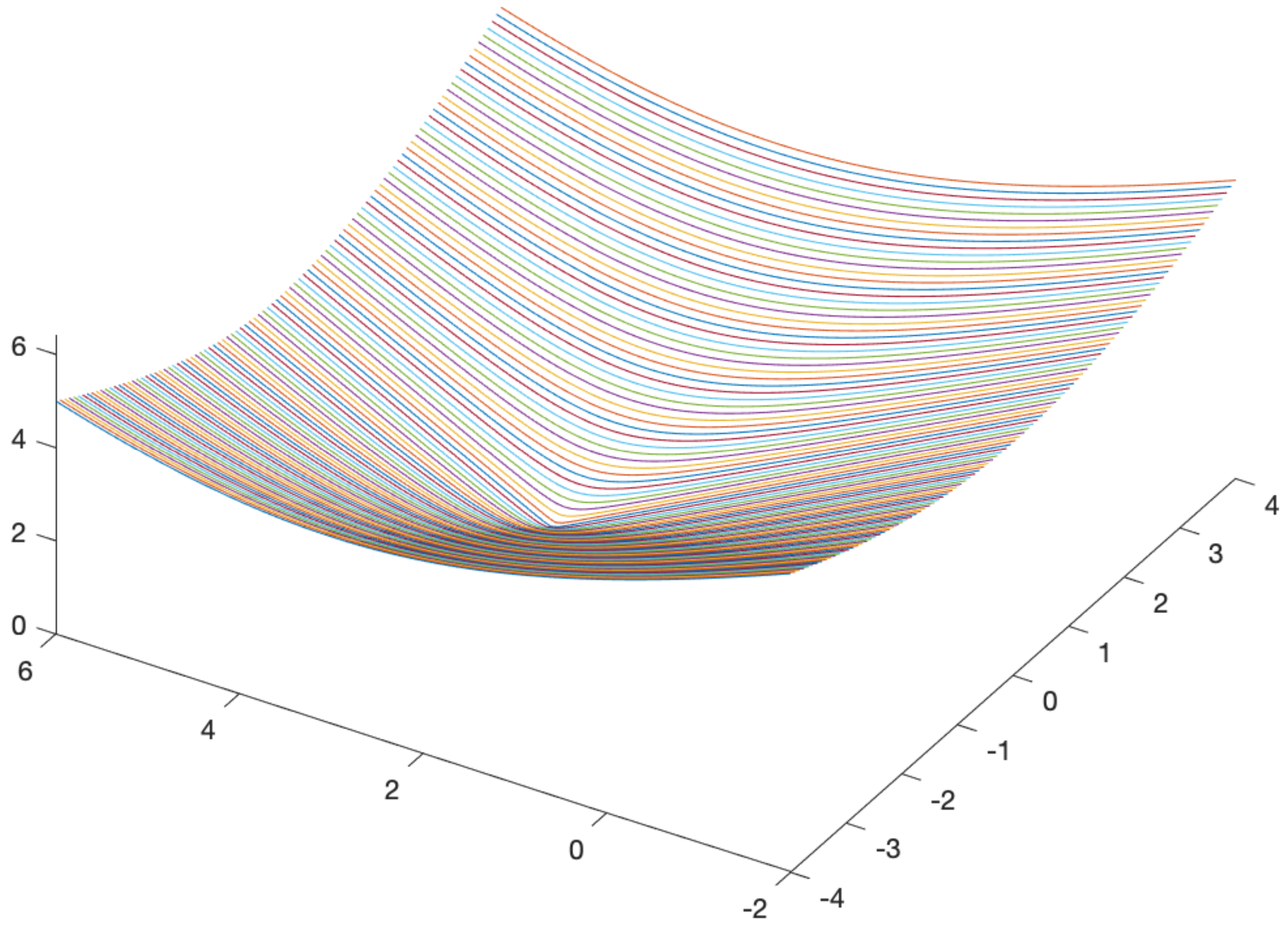
Attractive potential

The first step is to define an attractive potential field, e.g., proportional to the distance to the goal point (0,3)

$$U_a = C_a dist_g(x, y)$$

where

$$dist_g(x, y) = \sqrt{x^2 + (y-3)^2}$$

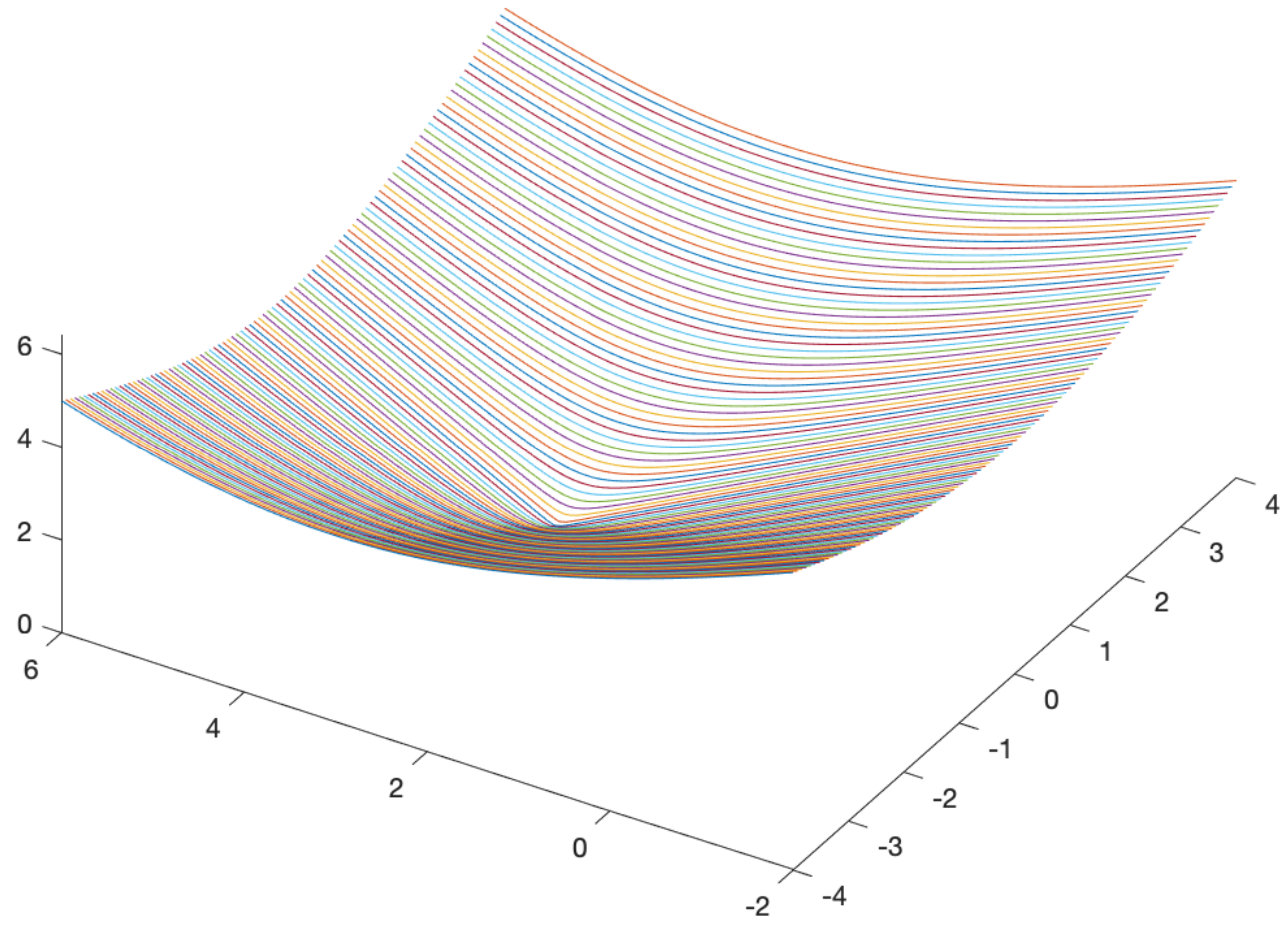


The attractive "force" is defined as the gradient of the potential with negative sign:

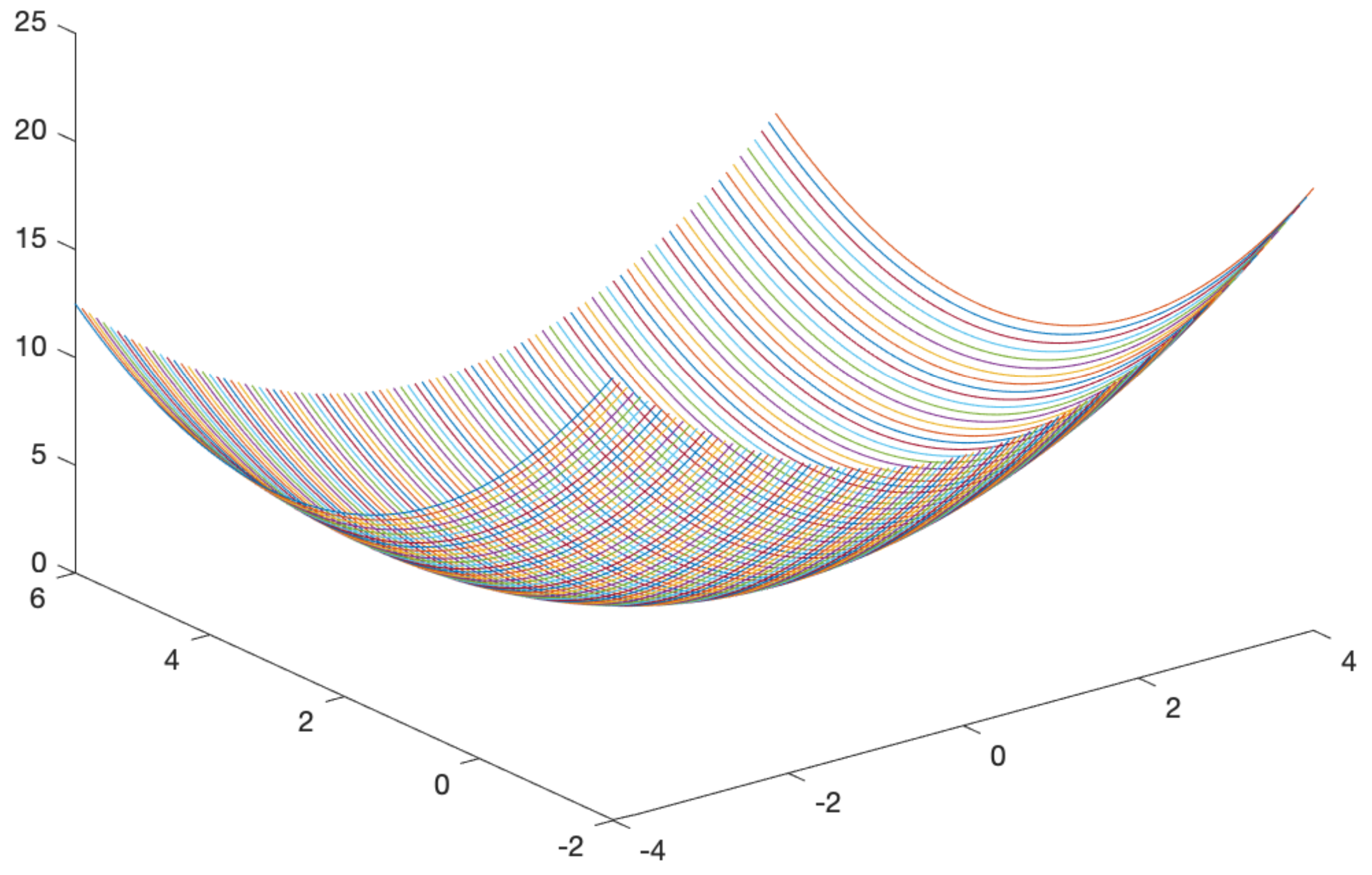
$$F_a = -\nabla U_a = -\frac{C_a}{\sqrt{x^2 + (y-3)^2}} \begin{pmatrix} x \\ y-3 \end{pmatrix}$$

Attractive potential

A drawback with this potential, is that the gradient is not continuous at the goal point, and a quadratic potential could be preferable.



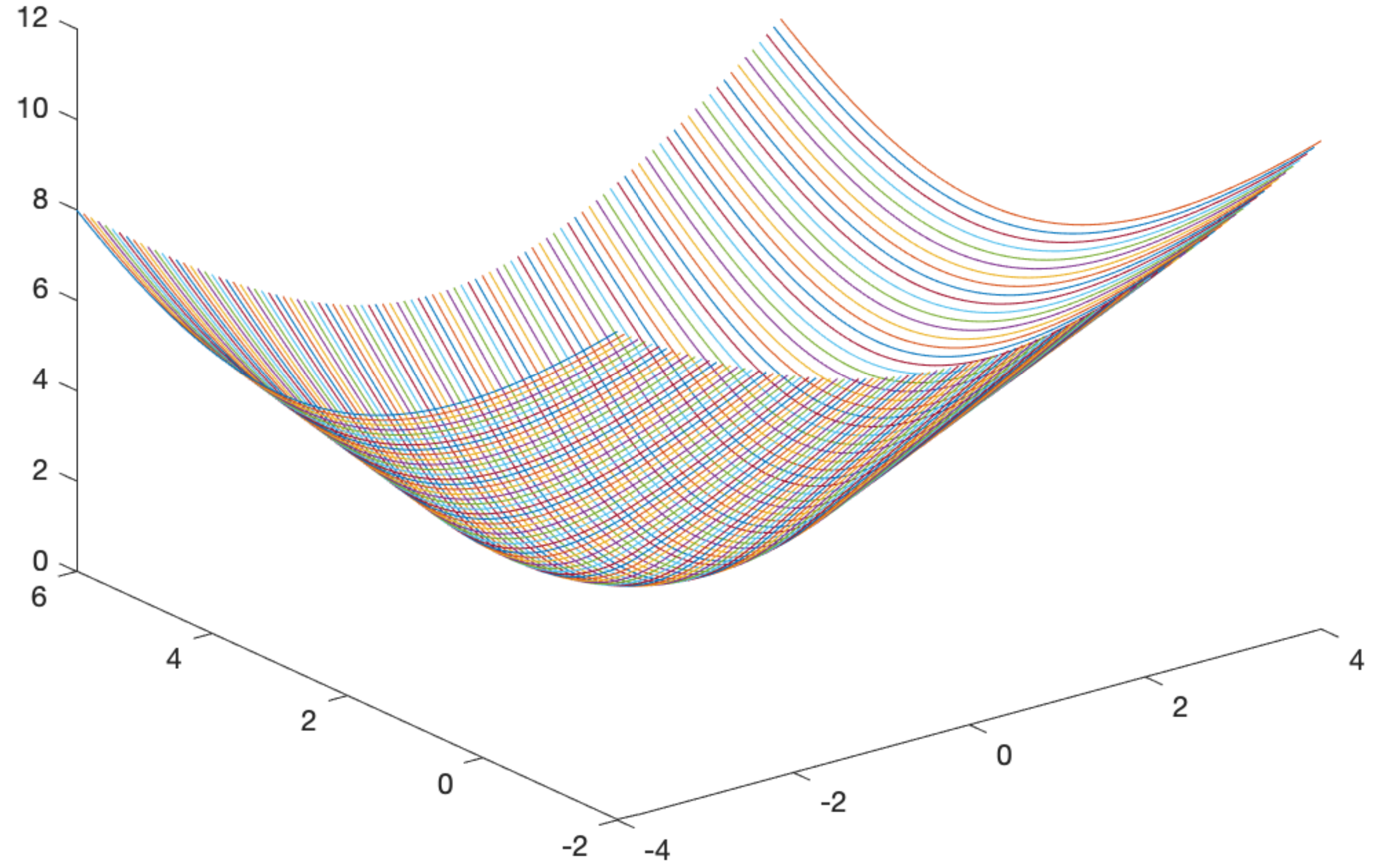
A drawback with a quadratic potential is that becomes large when the agent is far away from the goal, which can be difficult for the available actuators to handle.



Attractive Force, Composite Definition

A solution is to combine the potentials and use a quadratic potential near the goal and a linear further away from the goal:

$$U_a(x,y) = \begin{cases} \frac{C_a}{2} dist_g(x,y)^2 & \text{if } dist_g(x,y) \leq d^* \\ C_a d^* dist_g(x,y) - \frac{C_a}{2}(d^*)^2 & \text{if } dist_g(x,y) > d^* \end{cases}$$



Repulsive Force

To define the repulsive force the follow potential field is often used:

$$U_r(x, y) = \begin{cases} \frac{C_2}{2}\left(\frac{1}{dist_r(x, y)} - \frac{1}{q*}\right)^2 & \text{if } dist_r(x, y) \leq q* \\ 0 & \text{if } dist_r(x, y) > q* \end{cases}$$

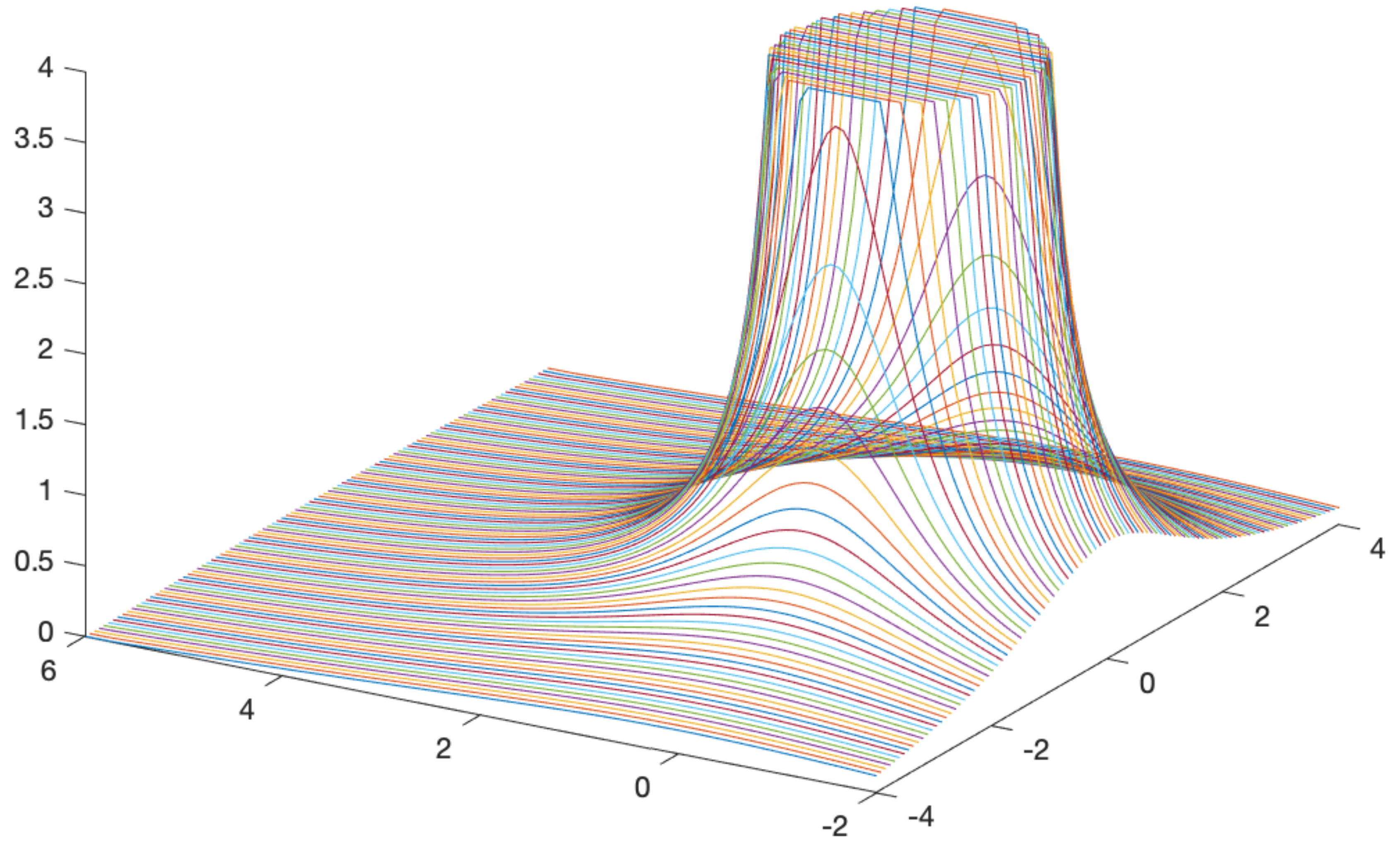
where

$$dist_r(x, y) = \min_{(x', y') \in O} \sqrt{(x - x')^2 + (y - y')^2}$$

is the distance from the point (x, y) to the closest point in the obstacle set O.

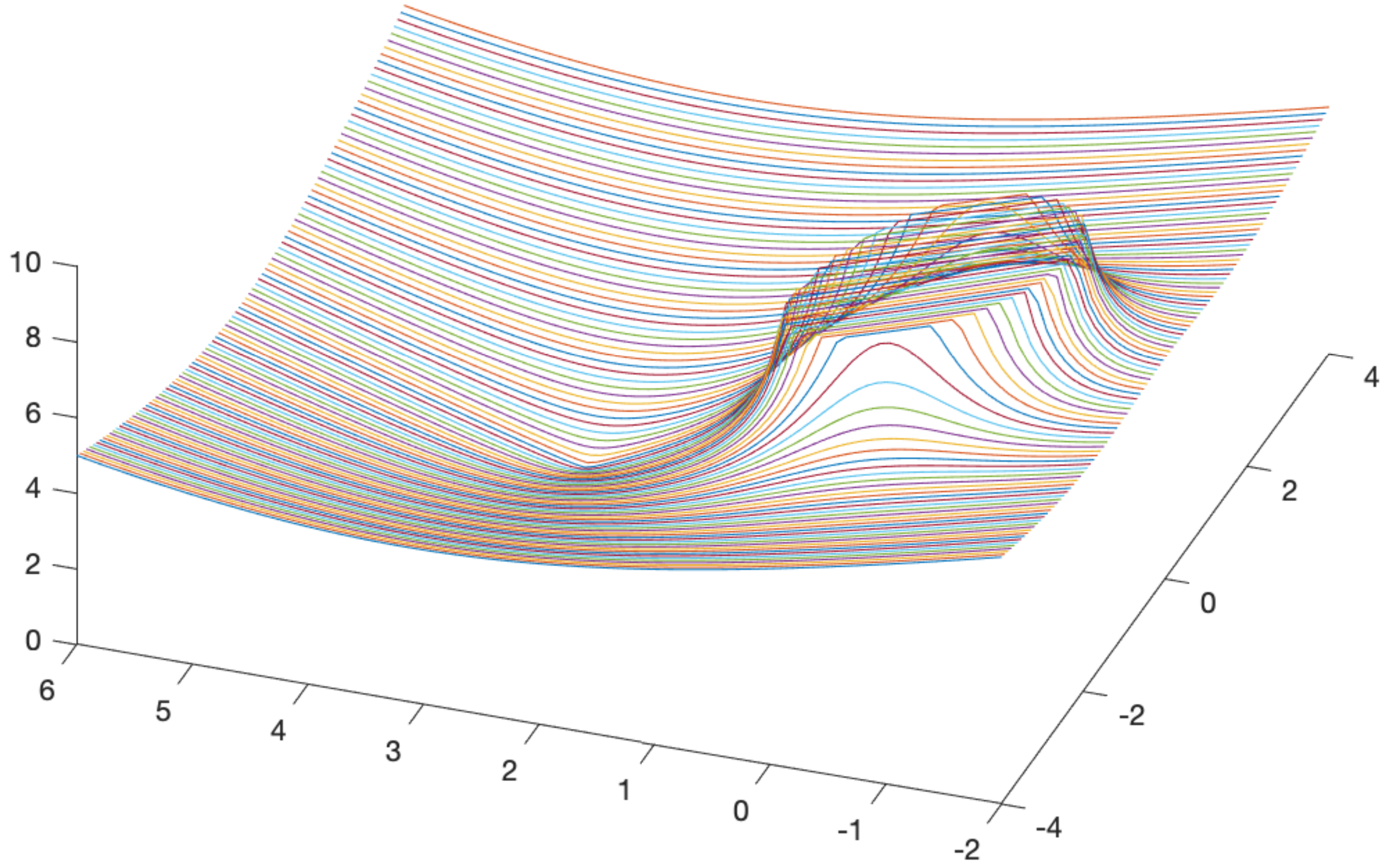
The repulsive force is defined by

$$F_r(x, y) = -\nabla U_r(x, y)$$



Artificial Potential Fields

The following figure shows the sum of the potential fields: $U_a(x, y) + U_r(x, y)$



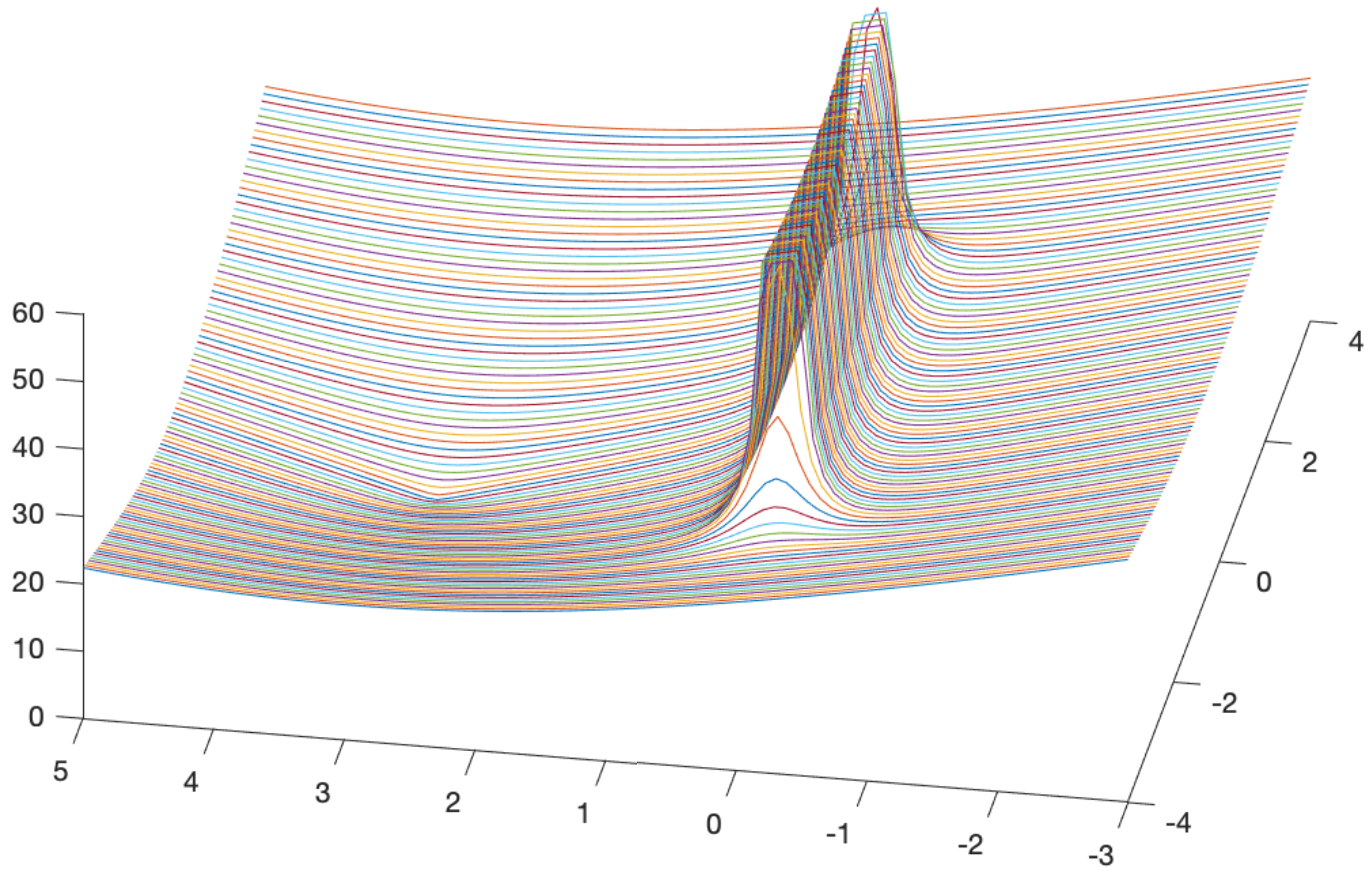
Artificial Potential Fields

Pros:

- Gradient can be calculated quickly
- Can handle moving obstacles and new obstacles that appear

Cons:

- The agent can be stucked in a local minima (see the figure to the right)



Artificial Potential Fields

Example how artificial potential field can be used to control the distance to the vehicle in front of you:

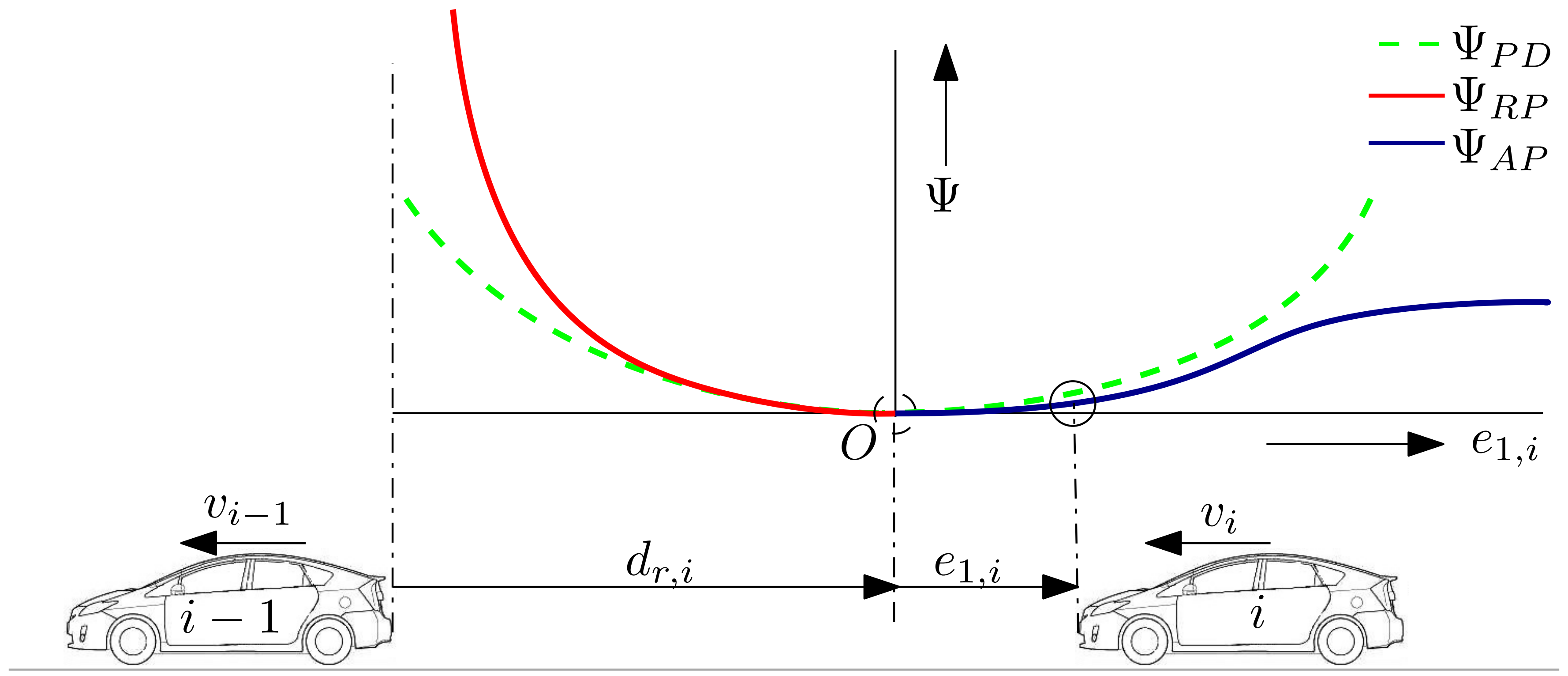


Figure 3.2: The desired repulsive and attractive potential (Ψ_{RP}, Ψ_{AP}), in comparison with a quadratic potential (Ψ_{PD})

From: Cooperative platoon maneuvering using Artificial Potential Fields,
K. Elferink, Master thesis, 2016, Eindhoven University of Technology